YORKSHIRE MATHS TUTOR

10. $$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

(*a*) Find the values of the constants a and b.

(2)

(*b*) Sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(3)

(*c*) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (*b*).

(2)

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(*d*) Find the set of possible values of k, giving your answer in surd form.

(4)

TOTAL FOR PAPER: 75 MARKS

END

completing the square

a) $x^2 + 2x + 3 = (x+1)^2 - 1 + 3$

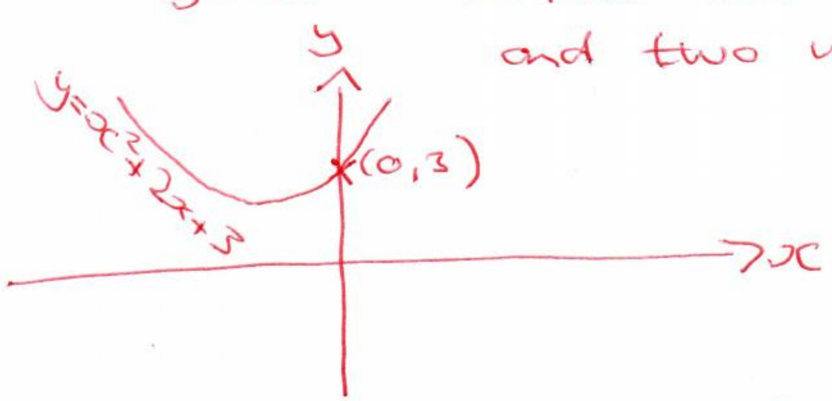
$= (x+1)^2 + 2$

in form $(x+a)^2 + b$

where $a = 1$, $b = 2$

b) $y = (x+1)^2 + 2$ is the graph of $y = x^2$ shifted one unit left and two units up

$x = 0$, $y = 3$



c) discriminant is $b^2 - 4ac$

$a = 1$, $b = 2$, $c = 3$

$b^2 - 4ac = (2^2) - 4(1)(3)$

$= 4 - 12$

$= -8$

As the discriminant is less than zero the curve does not cross x-axis

10 d) $x^2 + kx + 3 = 0$ no real roots

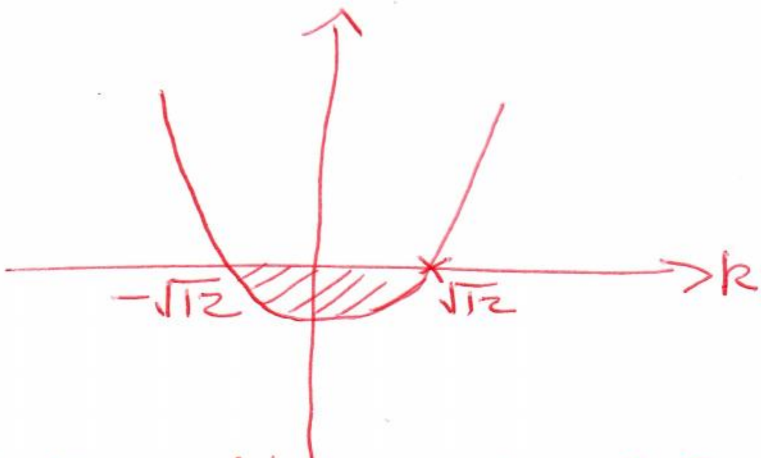
$\therefore b^2 - 4ac < 0$

$a = 1,\ b = k,\ c = 3$

$k^2 - 4(1)(3) < 0$

$k^2 - 12 < 0$

$(k + \sqrt{12})(k - \sqrt{12}) < 0$



From the diagram, the curve < 0 for

$-\sqrt{12} < k < \sqrt{12}$

JAN 2007

5. The equation $2x^2 - 3x - (k+1) = 0$, where k is a constant, has no real roots.

Find the set of possible values of k.

(4)

$a = 2, \; b = -3, \; c = -(k+1) = -k-1$

$b^2 - 4ac < 0$ for no real roots

$(-3)^2 - 4(2)(-k-1) < 0$

$9 + 8k + 8 < 0$

$8k < -9 - 8$

$8k < -17$

$k < -\frac{17}{8}$

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Q5

(Total 4 marks)

JAN 2008

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8. The equation

$$x^2 + kx + 8 = k$$

so $x^2 + kx + (8 - k) = 0$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

(b) Hence find the set of possible values of k.

(4)

a) No real solution so $b^2 - 4ac < 0$

$a = 1,\ b = k,\ c = 8 - k$

so $k^2 - 4 \times (1)(8 - k) < 0$

$k^2 - 32 + 4k < 0$

$\therefore\ k^2 + 4k - 32 < 0$ as required

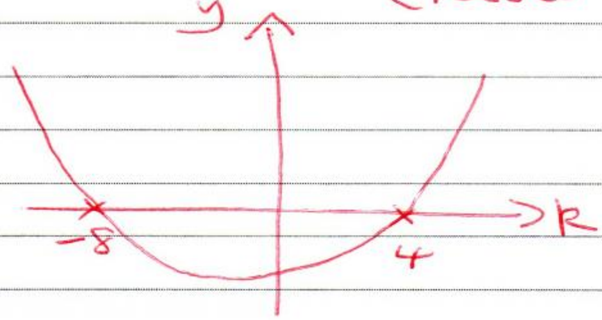
b) $(k - 4)(k + 8) < 0$

Find CRITICAL values for inequality

Either $k - 4 = 0$ or $k + 8 = 0$

$k = 4$ or $k = -8$

crosses k axis



We want values where y value is less than zero (below k axis)

$\therefore$ from graph

$-8 < k < 4$

JAN 2009

7. The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

Leave blank

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of k.

(4)

a)

For 2 real solutions, discriminant $b^2 - 4ac > 0$

$a = k$, $b = 4$, $c = 5 - k$

$4^2 - 4k(5-k) > 0$

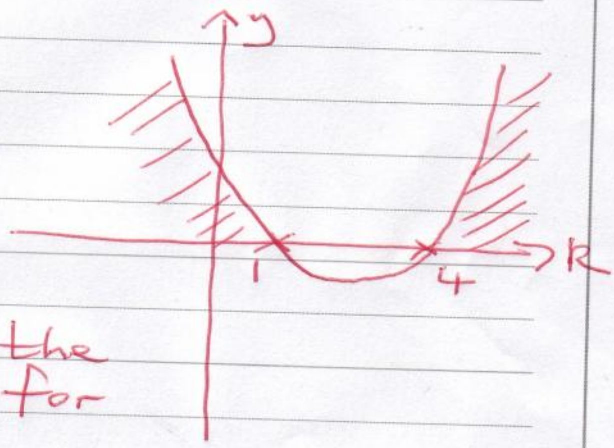
$16 - 20k + 4k^2 > 0$

$4(k^2 - 5k + 4) > 0$

$\therefore \quad k^2 - 5k + 4 > 0$ as required

b) $(k-4)(k-1) > 0$

Limiting values are $k = 4$ and $k = 1$



From the graph, the curve is > 0 for

$k < 1$ and $k > 4$

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8. The equation $x^2+(k-3)x+(3-2k)=0$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies

$$k^2+2k-3>0$$

(3)

(b) Find the set of possible values of k.

(4)

For 2 distinct, real roots the discriminat $b^2-4ac>0$

$a=1$, $b=(k-3)$, $c=(3-2k)$

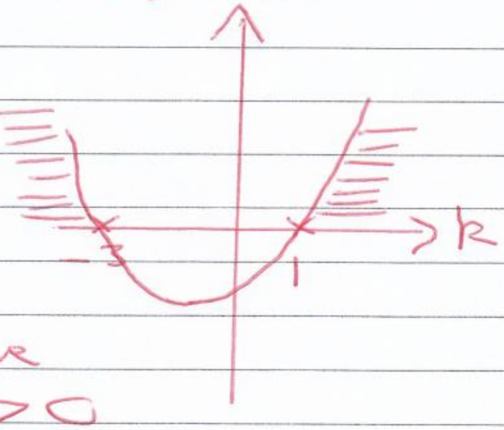
$\therefore\ (k-3)^2-4(1)(3-2k)>0$

$k^2-6k+9-12+8k>0$

$k^2+2k-3>0$ as required

b) $(k+3)(k-1)>0$

Critical values are $k=-3$, $k=1$



But we want values of k where curve for k function >0

$\therefore\ k<-3$ and $k>1$

C1 Jan 2012

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5. The curve C has equation $y = x(5 - x)$ and the line L has equation $2y = 5x + 4$

(a) Use algebra to show that C and L do not intersect.

(4)

(b) In the space on page 11, sketch C and L on the same diagram, showing the coordinates of the points at which C and L meet the axes.

(4)

a) $y = 5x - x^2$ ① $\quad 2y = 5x + 4$

$y = \frac{5}{2}x + 2$ ②

If intersect, ① = ②

$5x - x^2 = \frac{5}{2}x + 2$

$10x - 2x^2 = 5x + 4$

$0 = 2x^2 - 5x + 4$

$a = 2, b = -5, c = 4$

$b^2 - 4ac = 25 - 4 \times 2 \times 4$

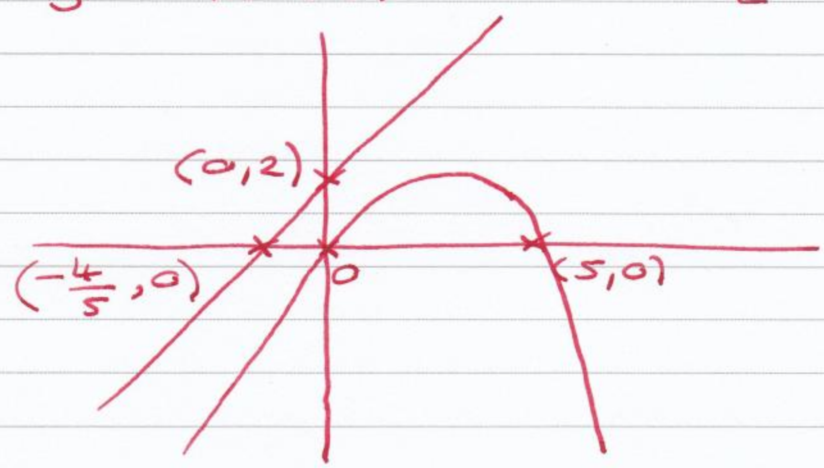
$= 25 - 32 = -7$

$b^2 - 4ac$ is discriminant < 0

$\therefore$ there are no real solutions

so C and L do not intersect

b) $y = x(5 - x) \qquad y = \frac{5}{2}x + 2$



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7. The equation $x^2 + kx + (k+3) = 0$, where k is a constant, has different real roots.

(a) Show that $k^2 - 4k - 12 > 0$.

(2)

(b) Find the set of possible values of k.

(4)

a) $x^2 + kx + (k+3) = 0$

$b^2 - 4ac > 0$ for different roots

$\therefore$ $a = 1$, $b = k$, $c = (k+3)$

$\therefore$ $k^2 - 4 \times 1 \times (k+3) > 0$

$\therefore$ $k^2 - 4k - 12 > 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant $b^2 - 4ac > 0$ for different real roots

b) Factorise to find the set of possible values for k

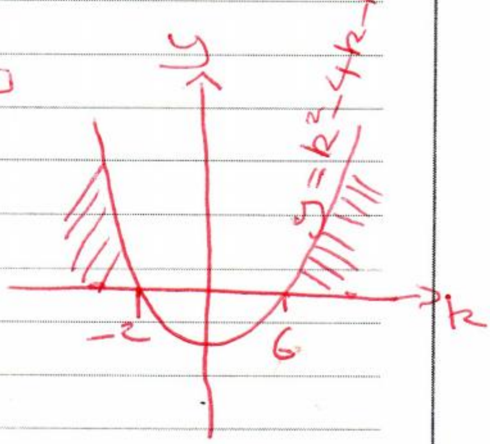
$\therefore$ $(k-6)(k+2) > 0$

For inequality — find CRITICAL VALUES

Critical values:

$\therefore$ $k - 6 = 0$ or $k + 2 = 0$

$\therefore$ $k = 6$ or $k = -2$



The graph is > 0 for y for k values < -2 and > 6

From the graph $k < -2$ or $k > 6$

8. Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$. **(2)**

(b) Hence find the set of possible values of q. **(3)**

a) $2qx^2 + qx - 1 = 0$

If no real roots the discriminant

$b^2 - 4ac < 0$

$\therefore q^2 - 4(2q)(-1) < 0$

$\therefore q^2 + 8q < 0$

$ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ from the Quadratic formula

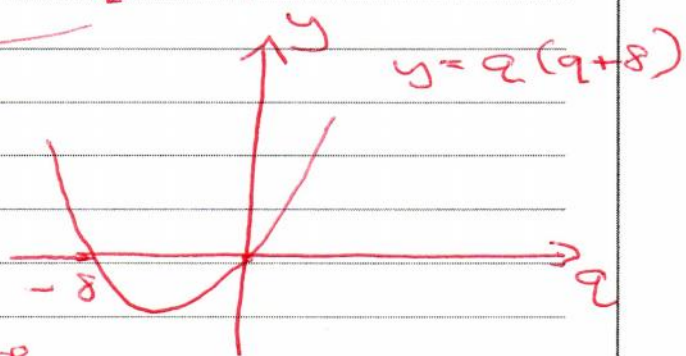
b) $q(q+8) < 0$

Find CRITICAL VALUES if there is an inequality

$q = 0$ or $q + 8 = 0$

$\therefore q = 0$ or $q = -8$

crosses q axis at $q = 0$ and $q = -8$



We want values where y value is less than zero (below q axis)

$\therefore$ from the graph

$-8 < q < 0$

6. The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(4)

For equal roots, $b^2 - 4ac = 0$

$a = 1$, $b = 3p$, $c = p$

$(3p)^2 - 4 \times (1)(p) = 0$

$9p^2 - 4p = 0$

$p(9p - 4) = 0$

Either $p = 0$ or $9p - 4 = 0$

$p = \frac{4}{9}$

As p is a non-zero constant

$p = \frac{4}{9}$

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8. The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(a) Find the value of p.

(4)

(b) For this value of p, solve the equation $x^2 + 2px + (3p + 4) = 0$.

(2)

For equal roots $b^2 - 4ac = 0$

$a = 1,\ b = 2p,\ c = 3p + 4$

$(2p)^2 - 4(1)(3p+4) = 0$

$4p^2 - 12p - 16 = 0$

$4(p^2 - 3p - 4) = 0$

$4(p-4)(p+1) = 0$

Either $p - 4 = 0$ or $p + 1 = 0$

$p = 4$ $\quad$ $p = -1$

As p is a positive, $p = 4$

b) $x^2 + 2px + (3p+4) = 0$

$x^2 + 8x + 12 + 4 = 0$

$x^2 + 8x + 16 = 0$

$(x+4)(x+4) = 0$

$x = -4$ or $x = -4$

May 2010

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4. (a) Show that $x^2+6x+11$ can be written as

$$(x+p)^2+q$$

where p and q are integers to be found.

(2)

(b) In the space at the top of page 7, sketch the curve with equation $y=x^2+6x+11$, showing clearly any intersections with the coordinate axes.

(2)

(c) Find the value of the discriminant of $x^2+6x+11$

(2)

a) $x^2+6x+11$ completing the square

$=(x+3)^2-9+11$

$=(x+3)^2+2$

in form $(x+p)^2+q$

where $p=3$ and $q=2$

b) $0=(x+3)^2+2$ Meets x-axis $(y=0)$

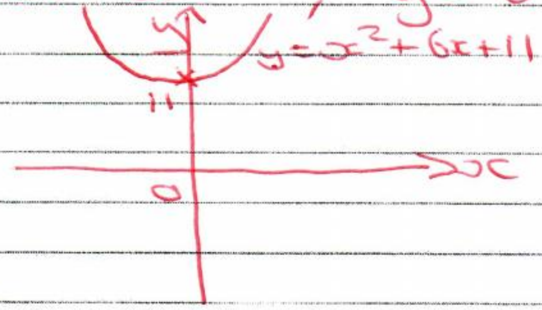
$(x+3)^2=-2$

$x+3=\sqrt{-2}$

No REAL solution for $\sqrt{-2}$

(ie does not meet x-axis)

When $x=0$, $y=0+0+11=11$



c) $x^2+6x+11$

$a=1$, $b=6$, $c=11$

b^2-4ac

$=36-4(1)(11)$

$=36-44=-8$

as $b^2-4ac<0$ there are no real roots, which is why curve does not intersect x-axis.

H 3 5 3 8 3 A 0 6 2 8

May 2011

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7. $$f(x) = x^2 + (k+3)x + k$$

where k is a real constant.

(a) Find the discriminant of $f(x)$ in terms of k.

(2)

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k+a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation $f(x) = 0$ has real roots.

(2)

a) $a = 1 \quad b = (k+3) \quad c = k$

$b^2 - 4ac = (k+3)^2 - 4 \times 1 \times k$

$= k^2 + 6k + 9 - 4k$

$= k^2 + 2k + 9$

b) $k^2 + 2k + 9$

$= (k+1)^2 - 1^2 + 9$

$= (k+1)^2 + 8$

where $a = 1$, $b = 8$

c) When we completed the square in part b)

$b^2 - 4ac = (k+1)^2 + 8$

as $(k+1)^2$ minimum value is 0, the minimum value of $b^2 - 4ac$ is 8

$\therefore b^2 - 4ac \geq 8$

$\therefore$ for all values of k $f(x) = 0$ has real roots

C1 MAY 2012

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8.
$$4x - 5 - x^2 = q - (x + p)^2$$

where p and q are integers.

(a) Find the value of p and the value of q. **(3)**

(b) Calculate the discriminant of $4x - 5 - x^2$ **(2)**

(c) On the axes on page 17, sketch the curve with equation $y = 4x - 5 - x^2$ showing clearly the coordinates of any points where the curve crosses the coordinate axes. **(3)**

a) swap values to other side of equation

$(x+p)^2 - q = x^2 - 4x + 5$

complete square for this

$(x-2)^2 - 2^2 + 5$

$= (x-2)^2 + 1$

where $p = -2$, $q = -1$

swap signs at end as we moved variables to other side

b) $-x^2 + 4x - 5$

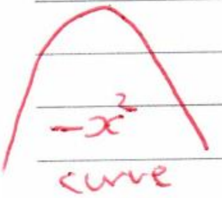
$a = -1,\ b = 4,\ c = -5$

$b^2 - 4ac = 4^2 - 4 \times -1 \times -5$

$= 16 - 20$

$= -4$

c) $4x - 5 - x^2 = -1 + (x-2)^2$



$-x^2$ curve this way up

intercept on y axis

goes through $(2, -1)$ maximum point

C1 MAY 2012

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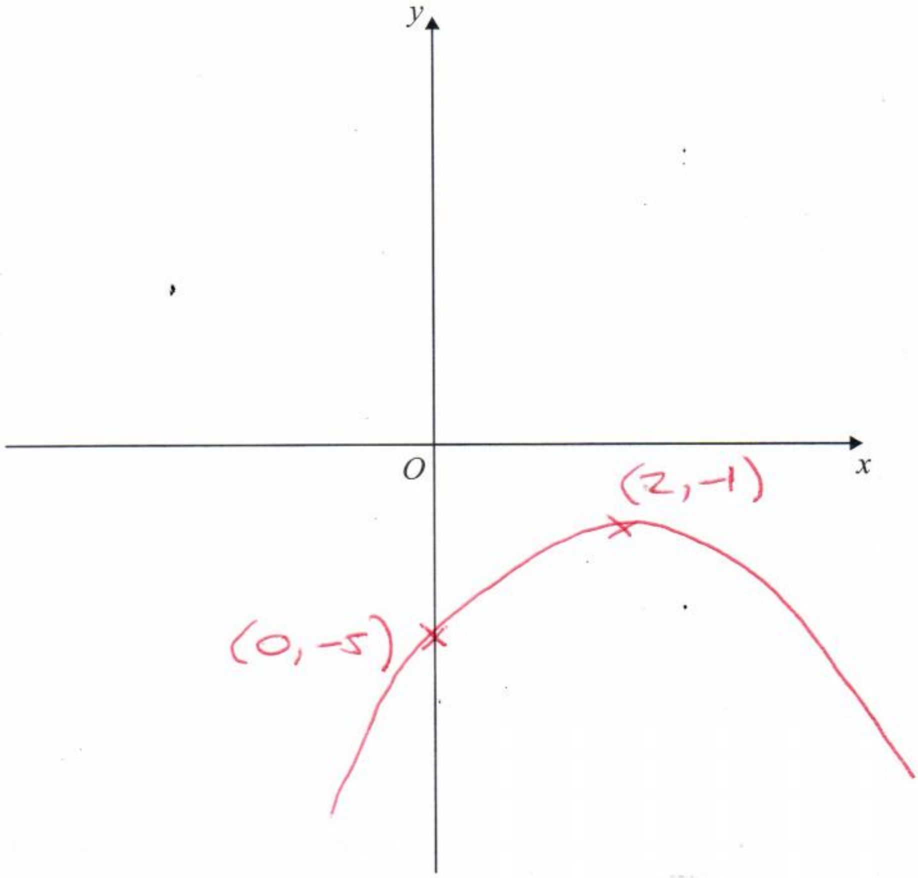
Question 8 continued



Q8

(Total 8 marks)

Turn over

C1 Jan 2013

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1. Factorise completely $x - 4x^3$

(3)

$x - 4x^3$

$= x(1 - 4x^2)$

$= x(1 + 2x)(1 - 2x)$

Q1

(Total 3 marks)

C1 Jan 2013

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10. $$4x^2 + 8x + 3 \equiv a(x+b)^2 + c$$

(a) Find the values of the constants a, b and c.

(3)

(b) On the axes on page 27, sketch the curve with equation $y = 4x^2 + 8x + 3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(4)

a) $4x^2 + 8x + 3$

$= 4\left(x^2 + 2x + \frac{3}{4}\right)$

$= 4\left[(x+1)^2 - 1 + \frac{3}{4}\right]$

$= 4\left[(x+1)^2 - \frac{1}{4}\right]$

$= 4(x+1)^2 - 1$

in form $a(x+b)^2 + c$

where $a = 4$, $b = 1$, $c = -1$

b) x^2 curve ∪

minimum when $x + 1 = 0$

$x = -1$

and $y = -1$

y-axis, when $x = 0$, $y = 4(0+1)^2 - 1$

$= 4 - 1 = 3$

x-axis when $0 = 4(x+1)^2 - 1$

$1 = 4(x+1)^2$

$\frac{1}{4} = (x+1)^2$

square root both side

$x + 1 = \frac{1}{2}$ or $x + 1 = -\frac{1}{2}$

$x = -\frac{1}{2}$ $\qquad$ $x = -\frac{3}{2}$

C1 Jan 2013

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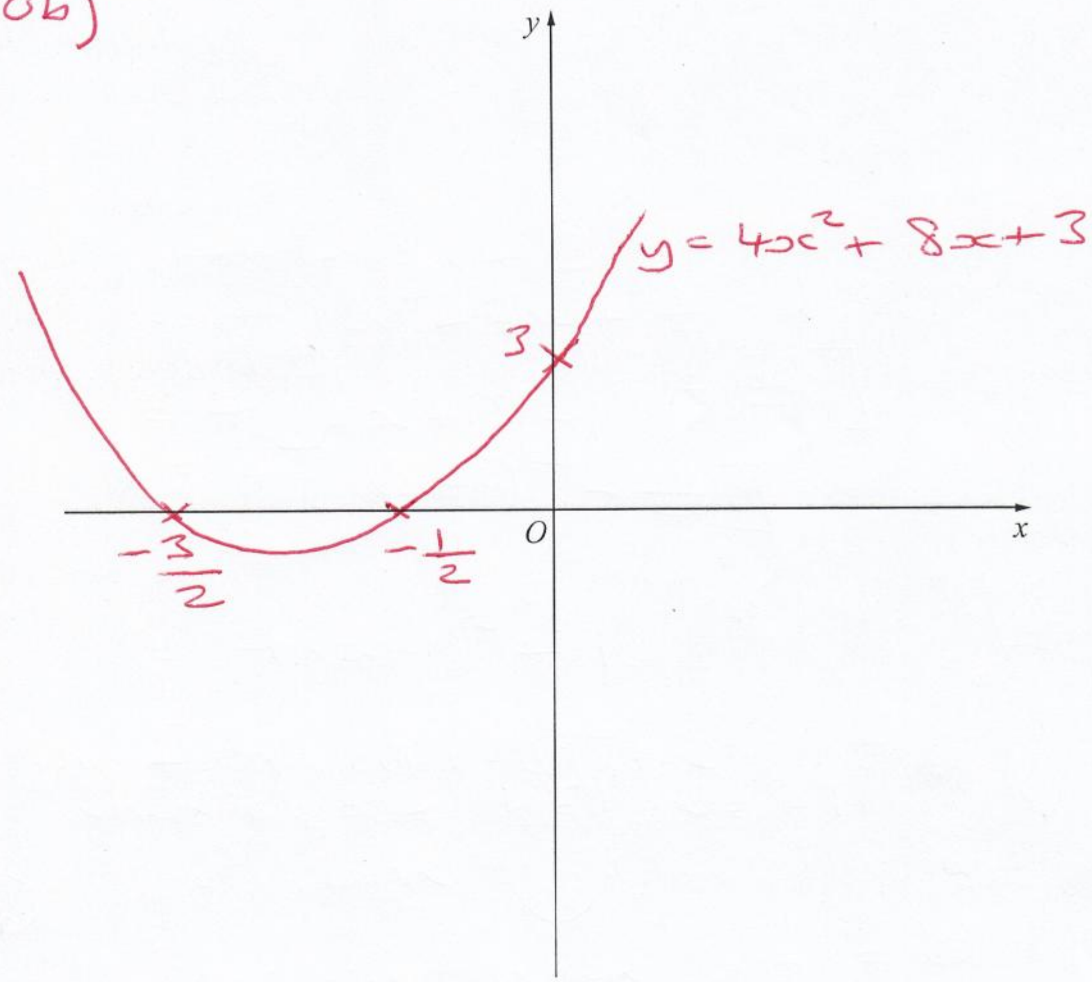
Question 10 continued

10b)



Turn over

C1 MAY 2013

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10. Given the simultaneous equations

$$2x + y = 1 \quad (1)$$
$$x^2 - 4ky + 5k = 0 \quad (2)$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0$$

(2)

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k.

(3)

(c) For this value of k, find the solution of the simultaneous equations.

(3)

a) (1) gives $y = 1 - 2x$
sub in (2)

$x^2 - 4k(1-2x) + 5k = 0$
$x^2 - 4k + 8kx + 5k = 0$
$x^2 + 8kx + k = 0$ (as required)

b) equal roots $b^2 - 4ac = 0$
$a = 1$, $b = 8k$, $c = k$

$(8k)^2 - 4 \times 1 \times k = 0$
$64k^2 - 4k = 0$
$4k(16k - 1) = 0$
Either $k = 0$ or $k = \frac{1}{16}$
as k is non-zero constant, $\underline{\underline{k = \frac{1}{16}}}$

c) $x^2 + \frac{8}{16}x + \frac{1}{16} = 0$ × through by 16

$16x^2 + 8x + 1 = 0$
$(4x+1)(4x+1) = 0$
$x = -\frac{1}{4}$ or $x = -\frac{1}{4}$
in (1) $y = 1 - 2 \times -\frac{1}{4} = \frac{3}{2}$

Solution to simultaneous equation
$x = -\frac{1}{4}$, $y = \frac{3}{2}$

$\underline{\underline{\left(-\frac{1}{4}, \frac{3}{2}\right)}}$

P 4 1 8 0 2 A 0 2 0 2 8