1. $$f(x) = x^3 + 3x^2 + 5.$$

Find

(a) $f''(x)$,

(3)

(b) $\int_1^2 f(x)\,dx$.

(4)

a) $f'(x) = 3x^2 + 6x$

$f''(x) = 6x + 6$

b) $\int_1^2 (x^3 + 3x^2 + 5)\,dx$

$= \left[\frac{x^4}{4} + x^3 + 5x\right]_1^2$

$= \left(\frac{2^4}{4} + 2^3 + 5(2)\right) - \left(\frac{1^4}{4} + 1^3 + 5(1)\right)$

$= (4 + 8 + 10) - (\frac{1}{4} + 1 + 5)$

$= 22 - 6\frac{1}{4}$

$= 15\frac{3}{4}$

Leave blank

8. A diesel lorry is driven from Birmingham to Bury at a steady speed of v kilometres per hour. The total cost of the journey, £C, is given by

$$C = \frac{1400}{v} + \frac{2v}{7}.$$

(a) Find the value of v for which C is a minimum. **(5)**

(b) Find $\frac{d^2C}{dv^2}$ and hence verify that C is a minimum for this value of v. **(2)**

(c) Calculate the minimum total cost of the journey. **(2)**

a) $C = 1400v^{-1} + \frac{2}{7}v$

$\frac{dC}{dv} = -1400v^{-2} + \frac{2}{7}$

for minimum value $\frac{dC}{dv} = 0$

$\therefore 0 = \frac{-1400}{v^2} + \frac{2}{7}$

$\frac{1400}{v^2} = \frac{2}{7}$

$v^2 = \frac{7 \times 1400}{2}$

$v^2 = 4900$

$v = 70 \text{ kmh}^{-1}$

b) $\frac{d^2C}{dv^2} = 2800v^{-3} = \frac{2800}{v^3}$

if $v = 70$, $\frac{d^2C}{dv^2} = \frac{2800}{70^3} = 8.16 \times 10^{-3}$

as $\frac{d^2C}{dv^2} > 0$ this value of C is a minimum

c) $C = \frac{1400}{70} + \frac{2 \times 70}{7} = 20 + 20 = 40$

$C = £40$

9a) continued

sub ② into ①

$$A = 3\cancel{x}^{1}\left(\frac{100}{x^{\cancel{2}1}}\right) + 2x^2$$

$$A = \frac{300}{x} + 2x^2$$

b) $A = 300x^{-1} + 2x^2$

$$\frac{dA}{dx} = -300x^{-2} + 4x$$

$$\frac{dA}{dx} = 4x - \frac{300}{x^2}$$

When A is stationary

$$\frac{dA}{dx} = 0$$

$$\therefore 4x - \frac{300}{x^2} = 0$$

$$\therefore 4x^3 - 300 = 0$$

$$\therefore 4x^3 = 300$$

$$\therefore x^3 = 75$$

$$x = \sqrt[3]{75} = 4.21716\ldots$$

$$x = 4.2\,m \quad (1dp)$$

c) $\frac{d^2A}{dx^2} = 600x^{-3} + 4$

$$\frac{d^2A}{dx^2} = \frac{600}{x^3} + 4$$

when $x = 4.21716\ldots$

$$\frac{d^2A}{dx^2} = \frac{600}{(4.21716\ldots)^3} + 4$$

$\frac{d^2A}{dx^2} = 12$ which is > 0

$\therefore x = 4.21716\ldots$ gives a minimum value for A

Leave blank

9.

Figure 4

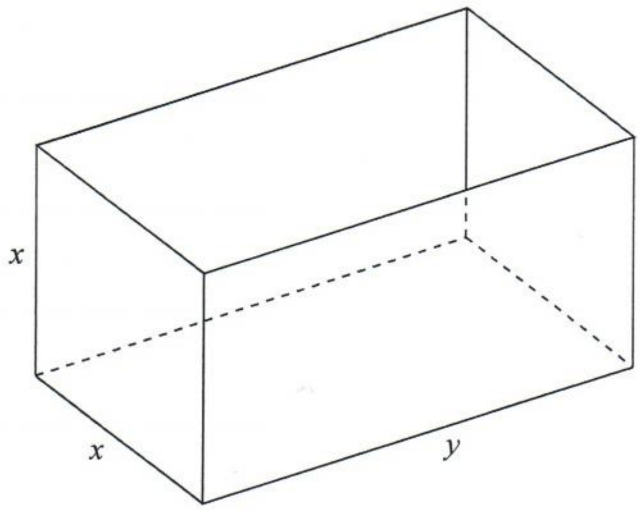


Figure 4 shows an open-topped water tank, in the shape of a cuboid, which is made of sheet metal. The base of the tank is a rectangle x metres by y metres. The height of the tank is x metres.

The capacity of the tank is 100 m^3.

(a) Show that the area A m^2 of the sheet metal used to make the tank is given by

$$A = \frac{300}{x} + 2x^2.$$

(4)

(b) Use calculus to find the value of x for which A is stationary.

(4)

(c) Prove that this value of x gives a minimum value of A.

(2)

(d) Calculate the minimum area of sheet metal needed to make the tank.

(2)

a) $A = xy + 2x^2 + 2xy$

$A = 3xy + 2x^2$ ①

Also V = area of cross section x height

$V = x^2y$

$\therefore 100 = x^2 y$

$\therefore y = \frac{100}{x^2}$ ②

9 d) when $x = 4.21716\ldots$ m

$$A = \frac{300}{4.21716\ldots} + 2(4.21716\ldots)^2$$

$$= 106.706\ldots$$

$$= 106.7 \text{ m}^2 \quad (1\text{dp})$$

Leave blank

9. The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$. **(4)**

(b) Hence show that $k = 12$. **(2)**

(c) Find the common ratio of this series. **(2)**

(d) Find the sum to infinity of this series. **(2)**

Geometric series

a) $a = k+4$ ① 1st term

$ar = k$ ② 2nd term

$ar^2 = 2k-15$ ③ 3rd term

① in ② gives

$$(k+4)r = k$$

$$r = \frac{k}{k+4}$$

sub for a and r^2 in ③ gives

$$ar^2 = 2k-15$$

$$\cancel{(k+4)}\left(\frac{k}{\cancel{k+4}}\right)\left(\frac{k}{k+4}\right) = 2k-15$$

$$\frac{k^2}{k+4} = 2k-15$$

$$k^2 = (2k-15)(k+4)$$

$$k^2 = 2k^2 + 8k - 15k - 60$$

$$0 = 2k^2 - k^2 - 7k - 60.$$

$k^2 - 7k - 60 = 0$ as required

b) $(k-12)(k+5) = 0$

Either $k-12=0$ or $k+5=0$

$k=12$ or $k=-5$

As k is positive

$k=12$

9c) ① gives $a = k + 4$

$a = 12 + 4 = 16$

in ② gives

$ar = k$

$r = \frac{k}{a} = \frac{12}{16} = \frac{3}{4}$

d) $S_\infty = \frac{a}{1-r} = \frac{16}{1-\frac{3}{4}} = \frac{16}{\frac{1}{4}}$

$= 64$

Leave blank

10. A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(a) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

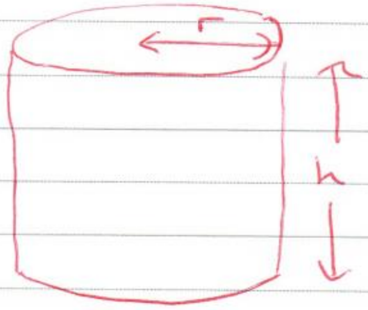
Given that r varies,

(b) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(c) Justify that the value of V you have found is a maximum.

(2)



$V = \pi r^2 h$ (1) – volume

a) Area of x-section = πr^2 (2 of these)

Area of curved surface = $2\pi r h$

Total area = $800 = 2\pi r^2 + 2\pi r h$

rearrange to get h

$h = \dfrac{800 - 2\pi r^2}{2\pi r}$

Put h from above in (1)

$V = \pi r^2 \times \left(\dfrac{\cancel{800}^{400} - \cancel{2}^{1}\pi r^2}{\cancel{2}\pi r} \right)$

$V = 400r - \pi r^3$ (as required)

10b) $V = 400r - \pi r^3$

$$\frac{dV}{dr} = 400 - 3\pi r^2$$

for maximum value $\frac{dV}{dr} = 0$

$$0 = 400 - 3\pi r^2$$
$$r^2 = \frac{400}{3\pi}$$
$$r = \sqrt{\frac{400}{3\pi}} = 6.5147002 \text{ cm}$$

$$V = 400r - \pi r^3$$
$$= 400 \times 6.5147002 - \pi (6.5147002)^3$$
$$= 1737.2534$$
$$= 1737 \text{ cm}^3 \text{ (nearest cm}^3\text{)}$$

c) $\frac{d^2V}{dr^2} = -6\pi r$

when $r = 6.5147002$

$$\frac{d^2V}{dr^2} = -6 \times \pi \times 6.5147002$$
$$= -122.7992$$

as $\frac{d^2V}{dr^2}$ is negative, the value of V is a maximum

Leave blank

9. The curve C has equation $y = 12\sqrt{(x)} - x^{\frac{3}{2}} - 10, \quad x > 0$

(a) Use calculus to find the coordinates of the turning point on C. **(7)**

(b) Find $\frac{d^2y}{dx^2}$. **(2)**

(c) State the nature of the turning point. **(1)**

a) $y = 12x^{\frac{1}{2}} - x^{\frac{3}{2}} - 10$

$\frac{dy}{dx} = \frac{1}{2} \times 12x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

$\frac{dy}{dx} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{6}{x^{\frac{1}{2}}} - \frac{3}{2}x^{\frac{1}{2}}$

At turning point $\frac{dy}{dx} = 0$

$0 = \frac{6}{x^{\frac{1}{2}}} - \frac{3}{2}x^{\frac{1}{2}}$

$0 = \frac{6}{\sqrt{x}} - \frac{3\sqrt{x}}{2}$

$\frac{3\sqrt{x}}{2} = \frac{6}{\sqrt{x}}$

$3\sqrt{x}\sqrt{x} = 2 \times 6$

$3x = 12$

$x = 4$

Put $x = 4$ in equation

$y = 12 \times \sqrt{4} - 4^{\frac{3}{2}} - 10$

$y = 24 - 8 - 10$

$y = 6$

Coordinates of turning point C are $(4, 6)$

9b) $\frac{dy}{dx} = 6x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

$\frac{d^2y}{dx^2} = -\frac{1}{2} \times 6x^{-\frac{3}{2}} - \frac{1}{2} \times \frac{3}{2} \times x^{-\frac{1}{2}}$

$\frac{d^2y}{dx^2} = -3x^{-\frac{3}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$

c) $\frac{d^2y}{dx^2} = -\frac{3}{x^{\frac{3}{2}}} - \frac{3}{4x^{\frac{1}{2}}}$

as $x = 4$

$\frac{d^2y}{dx^2} = -\frac{3}{4^{\frac{3}{2}}} - \frac{3}{4 \times 4^{\frac{1}{2}}}$

$= -\frac{3}{8} - \frac{3}{8} = -\frac{3}{4}$

As $\frac{d^2y}{dx^2} < 0$ the turning point is a maximum

Leave blank

10. The volume V cm^3 of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.
(4)

(b) Hence find the maximum volume of the box.
(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.
(2)

a) $V = 4x(25 - 10x + x^2)$
$V = 100x - 40x^2 + 4x^3$

$\frac{dV}{dx} = 100 - 80x + 12x^2$

b) $\frac{dV}{dx} = 0$ for maximum volume.
$0 = 12x^2 - 80x + 100$
$0 = 4(3x^2 - 20x + 25)$
$0 = 4(3x - 5)(x - 5)$
Either $x = \frac{5}{3}$ or $x = 5$

as $0 < x < 5$
$x = \frac{5}{3}$ so $V = 4 \times \frac{5}{3}\left(5 - \frac{5}{3}\right)^2$
$V = \frac{2000}{27}$ cm^3

c) $\frac{d^2V}{dx^2} = -80 + 24x$

at $x = \frac{5}{3}$
$\frac{d^2V}{dx^2} = -80 + \left(24 \times \frac{5}{3}\right)$
$= -40$

as $\frac{d^2V}{dx^2} < 0$ this is a maximum

H 3 5 4 0 3 A 0 2 6 2 8

C2 Jan 2012

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8.

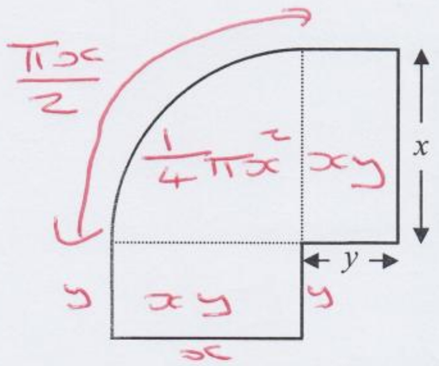


Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2,

(a) show that

$$y = \frac{16 - \pi x^2}{8x}$$

(3)

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x$$

(3)

(c) Use calculus to find the minimum value of P.

(5)

(d) Find the width of each rectangle when the perimeter is a minimum. Give your answer to the nearest centimetre.

(2)

a) area circle $= \pi x^2$

area quadrant $= \frac{1}{4}\pi x^2$

$4 = \frac{1}{4}\pi x^2 + xy + xy$

$4 = \frac{1}{4}\pi x^2 + 2xy$

$16 = \pi x^2 + 8xy$ × through by 4

$16 - \pi x^2 = 8xy$

$\frac{16 - \pi x^2}{8x} = y$ as required.

C2 Jan 2012

8b) arc length = $r\theta$ = $x \times \frac{\pi}{2}$

Perimeter = $\frac{\pi x}{2} + 4y + 2x$

from a) $y = \frac{16 - \pi x^2}{8x}$

$$P = \frac{\pi x}{2} + 4\left(\frac{16 - \pi x^2}{8x}\right) + 2x$$

$$P = \frac{\pi x}{2} + \frac{64}{8x} - \frac{4\pi x^2}{8x} + 2x$$

$$P = \frac{\pi x}{2} + \frac{8}{x} - \frac{\pi x}{2} + 2x$$

$$P = \frac{8}{x} + 2x \quad \text{(as required)}$$

8c) $P = 8x^{-1} + 2x$

$$\frac{dP}{dx} = -8x^{-2} + 2 = -\frac{8}{x^2} + 2$$

min when $\frac{dP}{dx} = 0$

$$0 = -\frac{8}{x^2} + 2$$

$$\frac{8}{x^2} = 2$$

$$x^2 = \frac{8}{2} = 4$$

$x = 2$ or $x = -2$ impossible as a length

d) $P = \frac{8}{2} + 2 \times 2 = 6 + 4 = 10\text{m}$

$$y = \frac{16 - \pi \times 2^2}{8 \times 2} = 0.2146 \text{ m}$$

$= 21.46$ cm

$= 21$ cm (to nearest cm)

Leave blank

2. Use calculus to find the exact value of $\int_1^2 \left(3x^2+5+\frac{4}{x^2}\right) dx$.

(5)

$$\int_1^2 (3x^2+5+4x^{-2})\,dx$$

$$= \left[\frac{3x^3}{3} + 5x + \frac{4x^{-1}}{-1}\right]_1^2$$

$$= \left[x^3 + 5x - \frac{4}{x}\right]_1^2$$

$$= \left(2^3+5(2)-\frac{4}{2}\right) - \left(1^3+5(1)-\frac{4}{1}\right)$$

$$= (8+10-2)-(1+5-4)$$

$$= 16 - 2$$

$$= 14$$

Q2

(Total 5 marks)

Leave blank

1. Evaluate $\int_1^8 \frac{1}{\sqrt{x}}\,dx$, giving your answer in the form $a+b\sqrt{2}$, where a and b are integers.

(4)

$$\int_1^8 x^{-\frac{1}{2}}\,dx$$

$$= \left[2x^{\frac{1}{2}}\right]_1^8$$

$$= \left(2\times 8^{\frac{1}{2}}\right) - \left(2\times 1^{\frac{1}{2}}\right)$$

$$= \left(2\times\sqrt{8}\right) - 2$$

$$= 2\times\sqrt{4}\times\sqrt{2} - 2$$

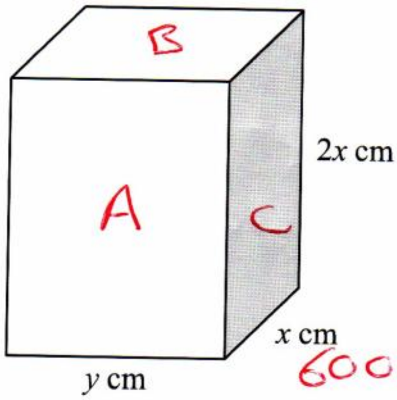
$$= 4\sqrt{2} - 2$$

$$= -2 + 4\sqrt{2}$$

Q1

(Total 4 marks)

10.



Area $A = 2xy$
$B = xy$
$C = 2x^2$

Total surface area $= 600$

$600 = 2(2xy + xy + 2x^2)$

① $600 = 2(3xy + 2x^2)$

Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(5)

(c) Justify that the value of V you have found is a maximum.

(2)

a) $V = 2x \times x \times y = 2x^2y$ ②

Rearranging ①

$600 = 6xy + 4x^2$

$6xy = 600 - 4x^2$

$y = \frac{600 - 4x^2}{6x}$

replace for y in ② gives

$V = \cancel{2}^{1}\cancel{x^2}^{1}\left(\frac{600 - 4x^2}{\cancel{6x}_{3}^{1}}\right)$

$V = \frac{600x}{3} - \frac{4x^3}{3}$

$V = 200x - \frac{4x^3}{3}$ as required

C2 May 2007

10b) $V = 200x - \frac{4}{3}x^3$

$$\frac{dV}{dx} = 200 - \frac{\cancel{3}^{1} \times 4}{\cancel{3}_{1}} x^2$$

$$\frac{dV}{dx} = 200 - 4x^2$$

At maximum point $\frac{dU}{dx} = 0$

$0 = 200 - 4x^2$

$4x^2 = 200$

$x^2 = \frac{200}{4}$

$x^2 = 50$

$x = \sqrt{50}$

$V = 200\sqrt{50} - \frac{4}{3}(\sqrt{50})^3$

$V = 942.809$

$V = 943\,cm^3$ (nearest cm^3)

10c) $\frac{d^2V}{dx^2} = -8x$

when $x = \sqrt{50}$

$\frac{d^2V}{dx^2} = -8 \times \sqrt{50} = -56.5685$

as this value is negative

we have proved that V is a maximum

1. Use calculus to find the value of

$$\int_1^4 \left(2x+3\sqrt{x}\right)\mathrm{d}x.$$

(5)

$\int_1^4 (2x+3x^{\frac{1}{2}})\,dx$

$= \left[\frac{2x^2}{2} + \frac{\cancel{2}\times\cancel{3}^{1}}{\cancel{3}_1}x^{\frac{3}{2}}\right]_1^4$

$= \left[x^2 + 2x^{\frac{3}{2}}\right]_1^4$

$= (4^2 + 2(4^{\frac{3}{2}})) - (1^2 + 2(1^{\frac{3}{2}}))$

$= (16 + 16) - (1 + 2)$

$= 32 - 3$

$= 29$

Leave blank

Q1

(Total 5 marks)

Leave blank

3.

$$y = x^2 - k\sqrt{x}, \quad \text{where } k \text{ is a constant.}$$

(a) Find $\dfrac{dy}{dx}$.

(2)

(b) Given that y is decreasing at $x = 4$, find the set of possible values of k.

(2)

a) $y = x^2 - k\sqrt{x}$

$y = x^2 - kx^{\frac{1}{2}}$

$\dfrac{dy}{dx} = 2x - \frac{1}{2}kx^{-\frac{1}{2}}$

b) $\dfrac{dy}{dx} < 0$ at $x=4$

$2x - \frac{1}{2}kx^{-\frac{1}{2}} < 0$ at $x=4$

$2\times4 - \frac{1}{2}k\times4^{-\frac{1}{2}} < 0$

$8 - \frac{1}{2}k \times \frac{1}{\sqrt{4}} < 0$

$8 - \frac{1}{4}k < 0$

$8 < \frac{1}{4}k$

$32 < k$

$k > 32$

May 2011

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8.

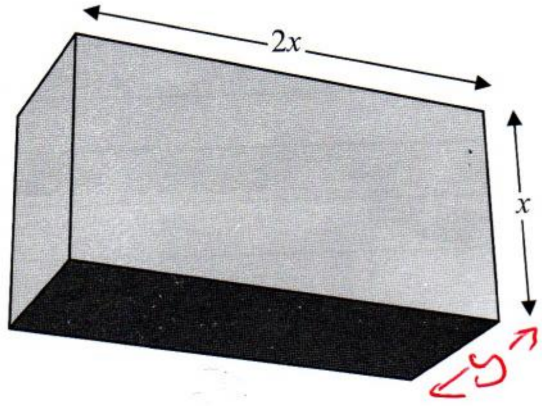

Figure 2

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 2.
The volume of the cuboid is 81 cubic centimetres.

(a) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}$$

(3)

(b) Use calculus to find the minimum value of L.

(6)

(c) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)

a) $V = \text{length} \times \text{width} \times \text{height}$

$81 = y \times 2x \times x$ (1)

$\therefore y = \dfrac{81}{2x^2}$

Total length of edges

$L = 4 \times 2x + 4 \times x + 4y$ (2)

Sub $y = \dfrac{81}{2x^2}$ in (2) from (1)

$L = 8x + 4x + 4 \times \dfrac{81}{2x^2}$

$L = 12x + \dfrac{162}{x^2}$ as required

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Question 8 continued

b) $L = 12x + 162x^{-2}$

$\frac{dL}{dx} = 12 - 324x^{-3}$

Minimum when $\frac{dL}{dx} = 0$

$0 = 12 - \frac{324}{x^3}$

$\frac{324}{x^3} = 12$

$x^3 = \frac{324}{12}$

$x = \sqrt[3]{\frac{324}{12}} = \sqrt{27} = 3$

Minimum value when $x = 3$

$L = 12 \times 3 + \frac{162}{3^2} = 54$ cm

c) $\frac{d^2L}{dx^2} = 972x^{-4} = \frac{972}{x^4}$

when $x = 3$

$\frac{d^2y}{dx^2} = \frac{972}{81} = 12$

as $\frac{d^2y}{dx^2}$ is positive

this is a minimum

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8.

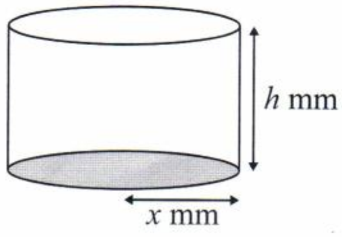

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(a) express h in terms of x,

(1)

(b) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$

(3)

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum.

(5)

(d) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)

(e) Show that this value of A is a minimum.

(2)

a) $V = \pi r^2 h$

$60 = \pi \times x^2 \times h$

$h = \dfrac{60}{\pi x^2}$

b) Area top $= \pi x^2$

Area base $= \pi x^2$ circumference

Curved area $= (\pi \times 2x) \times h$

so

$A = \pi x^2 + \pi x^2 + 2\pi x h$

substitute $h = \dfrac{60}{\pi x^2}$ from a)

$A = 2\pi x^2 + \dfrac{2\cancel{\pi}\cancel{x} \times 60}{\cancel{\pi} x^{\cancel{2}}}$

C2 May 2012

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Question 8 continued

$A = 2\pi x^2 + \frac{120}{x}$ as required

c) $A = 2\pi x^2 + 120x^{-1}$

$\frac{dA}{dx} = 4\pi x - 120x^{-2}$

at minimum point $\frac{dA}{dx} = 0$

$0 = 4\pi x - \frac{120}{x^2}$

$\frac{120}{x^2} = 4\pi x$

$\frac{120}{4\pi} = x^3$

$x = \sqrt[3]{\frac{120}{4\pi}}$

$x = 2.1215688$

$x = 2.12$ mm (3sf)

d) put $x = 2.1215688$ in equation for A

$A = 2 \times \pi \times 2.1215688^2 + \frac{120}{2.1215688}$

$A = 84.842875$

$A = 85$ mm^2 (nearest integer)

e) $\frac{d^2A}{dx^2} = 4\pi + 240x^{-3}$

$= 4\pi + \frac{240}{x^3}$

when $x = 2.1215688$, $\frac{d^2A}{dx^2} = 37.699113$

As $\frac{d^2A}{dx^2} > 0$ this value of A is a minimum

C2 Jan 2013

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8. The curve C has equation $y = 6 - 3x - \frac{4}{x^3}$, $x \neq 0$

(a) Use calculus to show that the curve has a turning point P when $x = \sqrt{2}$ **(4)**

(b) Find the x-coordinate of the other turning point Q on the curve. **(1)**

(c) Find $\frac{d^2y}{dx^2}$. **(1)**

(d) Hence or otherwise, state with justification, the nature of each of these turning points P and Q. **(3)**

a) $y = 6 - 3x - 4x^{-3}$

$\frac{dy}{dx} = -3 + 12x^{-4} = -3 + \frac{12}{x^4}$

Turning point when $\frac{dy}{dx} = 0$

$0 = -3 + \frac{12}{x^4}$

$3 = \frac{12}{x^4}$

$x^4 = \frac{12}{3}$

$x^4 = 4$

$x = \sqrt[4]{4} = \pm\sqrt{2}$

So P does have a turning point when $x = \sqrt{2}$

b) $x = -\sqrt{2}$ (from a)

c) $\frac{d^2y}{dx^2} = -48x^{-5} = -\frac{48}{x^5}$

d) at $x = \sqrt{2}$, $\frac{d^2y}{dx^2} = \frac{-48}{4\sqrt{2}} < 0$ maximum point

at $x = -\sqrt{2}$, $\frac{d^2y}{dx^2} = \frac{-48}{-4\sqrt{2}} > 0$ minimum point

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Leave blank

6.

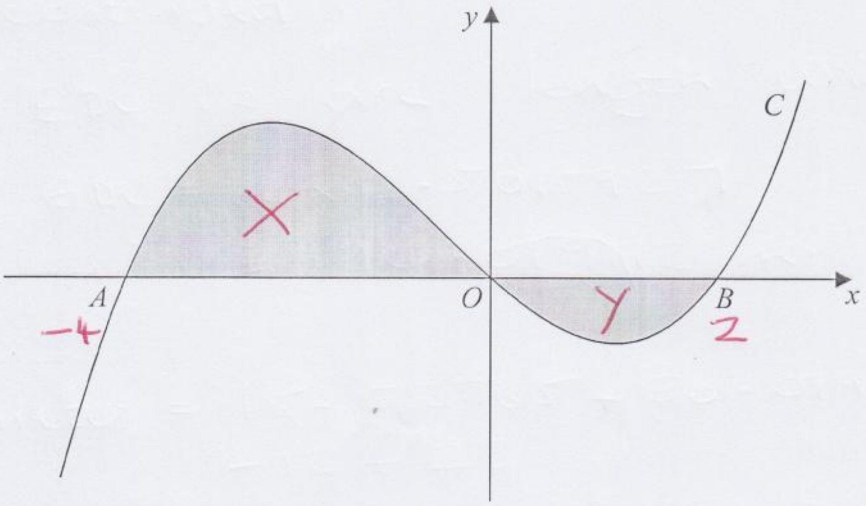

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

a) $A(-4,0)$ $\quad B(2,0)$

b) $y = x(x+4)(x-2)$

$y = x(x^2+2x-8)$

$y = x^3+2x^2-8x$

$\int_{-4}^{0} (x^3+2x^2-8x)\,dx$ (area X)

$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{-4}^{0}$

$= (0 - 0 - 0) - \left(\frac{256}{4} + \frac{128}{3} - 64\right)$

$= \frac{128}{3}$

6b) continued

$$\int_0^2 (x^3 + 2x^2 - 8x)\,dx \qquad \text{(area Y)}$$

$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - 4x^2\right]_0^2$$

$$= \left(4 + \frac{16}{3} - 16\right) - (0 - 0 - 0)$$

$$= -\frac{20}{3} \quad \text{(make this positive)}$$

$$\text{Total area} = \frac{128}{3} + \frac{20}{3} = \underline{\underline{\frac{148}{3} \text{ square units}}}$$

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9. The curve with equation

$$y = x^2 - 32\sqrt{(x)} + 20, \qquad x > 0$$

has a stationary point P.

Use calculus

(a) to find the coordinates of P,

(6)

(b) to determine the nature of the stationary point P.

(3)

a) $y = x^2 - 32x^{\frac{1}{2}} + 20$

$\frac{dy}{dx} = 2x - 16x^{-\frac{1}{2}}$

at P $\frac{dy}{dx} = 0$

$0 = 2x - \frac{16}{x^{\frac{1}{2}}}$

$\frac{16}{x^{\frac{1}{2}}} = 2x$

$\frac{16}{2} = x \times x^{\frac{1}{2}}$

$8 = x^{\frac{3}{2}}$ (square both sides)

$64 = x^3$

$x = \sqrt[3]{64} = 4$

at $x = 4$, $y = 4^2 - 32\sqrt{4} + 20 = 16 - 64 + 20 = -28$

Coordinates of P are $\underline{\underline{(4, -28)}}$

b) $\frac{d^2y}{dx^2} = 2 + 8x^{-\frac{3}{2}} = 2 + \frac{8}{x^{\frac{3}{2}}}$

at $x = 4$, $\frac{d^2y}{dx^2} = 2 + \frac{8}{(4^{\frac{1}{2}})^3}$

$= 2 + \frac{8}{8} = 3$

as $\frac{d^2y}{dx^2} > 0$ $P(-4, 28)$ is a minimum point