4. Given that $y = 2x^2 - \frac{6}{x^3}$, $x \neq 0$,

(*a*) find $\frac{dy}{dx}$,

(2)

(*b*) find $\int y \, dx$.

(3)

---

a) $y = 2x^2 - \frac{6}{x^3}$ $\quad$ $y = 2x^2 - 6x^{-3}$

$\frac{dy}{dx} = 4x + 18x^{-4}$

b) $\int y \, dx$

$= \int (2x^2 - 6x^{-3})\, dx$

$= \frac{2x^3}{3} - \frac{6x^{-2}}{-2} + c$

$= \frac{2}{3}x^3 + \frac{3}{x^2} + c$

Leave blank

1. Given that

$$y = 4x^3 - 1 + 2x^{\frac{1}{2}}, \quad x > 0,$$

find $\dfrac{dy}{dx}$.

(4)

$y = 4x^3 - 1 + 2x^{\frac{1}{2}}$

$\dfrac{dy}{dx} = 12x^2 + x^{-\frac{1}{2}}$

Q1

(Total 4 marks)

Leave blank

1. Find $\int(3x^2 + 4x^5 - 7)\,dx$.

(4)

$\int(3x^2 + 4x^5 - 7)\,dx$

$= \frac{3x^3}{3} + \frac{4x^6}{6} - 7x + c$

$= x^3 + \frac{2x^6}{3} - 7x + c$

Q1

(Total 4 marks)

Leave blank

5. (a) Write $\dfrac{2\sqrt{x}+3}{x}$ in the form $2x^{p}+3x^{q}$ where p and q are constants. **(2)**

Given that $y = 5x-7+\dfrac{2\sqrt{x}+3}{x}$, $x > 0$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term. **(4)**

a) $\dfrac{2\sqrt{x}+3}{x} = \dfrac{2x^{\frac{1}{2}}}{x} + \dfrac{3}{x}$

$= 2x^{-\frac{1}{2}} + 3x^{-1}$

(where $p=-\frac{1}{2}$, $q=-1$)

b) $y = 5x - 7 + \dfrac{2\sqrt{x}+3}{x}$

$y = 5x - 7 + 2x^{-\frac{1}{2}} + 3x^{-1}$

$\dfrac{dy}{dx} = 5 - x^{-\frac{3}{2}} - 3x^{-2}$

Leave blank

2. Find $\int(12x^5 - 8x^3 + 3)\,dx$, giving each term in its simplest form.

(4)

$$\int(12x^5 - 8x^3 + 3)\,dx$$

$$= \frac{12x^6}{6} - \frac{8x^4}{4} + 3x + c$$

$$= 2x^6 - 2x^4 + 3x + c$$

Q2

(Total 4 marks)

Leave blank

6. Given that $\dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q.

(2)

Given that $y = 5x^4 - 3 + \dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term.

(4)

a) $\dfrac{2x^2}{x^{\frac{1}{2}}} - \dfrac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$

$= 2x^{2-\frac{1}{2}} - x^{\frac{3}{2}-\frac{1}{2}}$

$= 2x^{\frac{3}{2}} - x$

This is in the form $2x^p - x^q$

where $p = \frac{3}{2}$ and $q = 1$

b) using answer to a)

$y = 5x^4 - 3 + 2x^{\frac{3}{2}} - x$

$\dfrac{dy}{dx} = 20x^3 + \frac{3}{2} \times 2x^{\frac{1}{2}} - 1$

$\dfrac{dy}{dx} = 20x^3 + 3x^{\frac{1}{2}} - 1$

Leave blank

7. The curve with equation $y = f(x)$ passes through the point $(-1, 0)$.

Given that

$$f'(x) = 12x^2 - 8x + 1$$

find $f(x)$.

(5)

$f'(x) = 12x^2 - 8x + 1$

Integrate to get $f(x)$

$f(x) = \frac{12x^3}{3} - \frac{8x^2}{2} + x + c$

when $x = -1$, $f(x) = 0$

$0 = 4(-1)^3 - 4(-1)^2 + (-1) + c$

$0 = -4 - 4 - 1 + c$

$c = 9$

$f(x) = 4x^3 - 4x^2 + x + 9$

C1 Jan 2012

Leave blank

1. Given that $y = x^4 + 6x^{\frac{1}{2}}$, find in their simplest form

(a) $\dfrac{dy}{dx}$

(3)

(b) $\int y\,dx$

(3)

a) $y = x^4 + 6x^{\frac{1}{2}}$

$\dfrac{dy}{dx} = 4x^3 + 3x^{-\frac{1}{2}}$

b) $\int y\,dx = \int (x^4 + 6x^{\frac{1}{2}})\,dx$

$= \dfrac{x^5}{5} + \dfrac{2}{3} \times 6 \times x^{\frac{3}{2}} + c$

$= \dfrac{x^5}{5} + 4x^{\frac{3}{2}} + c$

Leave blank

3. Given that $y = 3x^2 + 4\sqrt{x}, \quad x > 0$, find

(a) $\dfrac{dy}{dx}$, **(2)**

(b) $\dfrac{d^2y}{dx^2}$, **(2)**

(c) $\int y\,dx$. **(3)**

a) $y = 3x^2 + 4\sqrt{x}$

$y = 3x^2 + 4x^{\frac{1}{2}}$

$\therefore \dfrac{dy}{dx} = 6x + 2x^{-\frac{1}{2}}$

if $y = ax^n$ then $\dfrac{dy}{dx} = anx^{n-1}$

b) $\dfrac{d^2y}{dx^2} = 6 - x^{-\frac{3}{2}}$

c) $\int y\,dx = \int (3x^2 + 4x^{\frac{1}{2}})\,dx$

$= \dfrac{3x^3}{3} + \dfrac{4x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= x^3 + \dfrac{8}{3}x^{\frac{3}{2}} + c$

$\int ax^n\,dx = \dfrac{ax^{n+1}}{n+1}$

Leave blank

1. Find $\int (2 + 5x^2)\,dx$.

(3)

$\int (2+5x^2)\,dx$

$= 2x + \dfrac{5x^3}{3} + c$

$\int a\,dx = ax$

$\int ax^n\,dx$

$= \dfrac{ax^{n+1}}{n+1}$

Q1

(Total 3 marks)

Leave blank

4. $$f(x) = 3x + x^3, \qquad x > 0.$$

(a) Differentiate to find $f'(x)$.

(2)

Given that $f'(x) = 15$,

(b) find the value of x.

(3)

a) $f(x) = 3x + x^3$

$f'(x) = 3 + 3x^2$

b) when $f'(x) = 15$

$\therefore\ 3 + 3x^2 = 15$

$\therefore\ 3x^2 = 12$

$x^2 = 4$

$x = \sqrt{4}$

$\therefore\ x = \pm 2$

since $x > 0$ (in question)

$x = 2$

Leave blank

3. Given that $y = 2x^3 + \dfrac{3}{x^2}$, $x \neq 0$, find

(a) $\dfrac{dy}{dx}$ **(3)**

(b) $\int y\,dx$, simplifying each term. **(3)**

a) $y = 2x^3 + 3x^{-2}$

$\dfrac{dy}{dx} = 6x^2 - 6x^{-3}$

b) $\int y \; dx$

$= \int (2x^3 + 3x^{-2})\,dx$

$= \dfrac{2x^4}{4} + \dfrac{3x^{-1}}{-1} + c$

$= \frac{1}{2}x^4 - 3x^{-1} + c$

Leave blank

9.

$$f(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}}, \quad x > 0$$

(a) Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found. **(3)**

(b) Find $f'(x)$. **(3)**

(c) Evaluate $f'(9)$. **(2)**

a) $f(x) = \dfrac{(3-4\sqrt{x})^2}{\sqrt{x}}$

$= \dfrac{9 - 24\sqrt{x} + 16x}{x^{\frac{1}{2}}}$

$f(x) = 9x^{-\frac{1}{2}} - 24 + 16x^{\frac{1}{2}}$

$= 9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24$

where $A = 16$ and $B = -24$

b) $f'(x) = -\frac{9}{2}x^{-\frac{3}{2}} + \frac{1}{2}\cdot 16x^{-\frac{1}{2}}$

$= -\frac{9}{2}x^{-\frac{3}{2}} + 8x^{-\frac{1}{2}}$

c) $f'(9) = -\frac{9}{2}\cdot 9^{-\frac{3}{2}} + 8\cdot 9^{-\frac{1}{2}}$

$= -\dfrac{9}{2\times 9^{\frac{3}{2}}} + \dfrac{8}{9^{\frac{1}{2}}}$

$= -\dfrac{\cancel{9}^{1}}{2\times\cancel{27}_{3}} + \dfrac{8}{3}$

$= -\frac{1}{6} + \frac{8}{3}$

$= -\frac{1}{6} + \frac{16}{6}$

$= \frac{15}{6} = \frac{5}{2}$

$= 2\frac{1}{2}$

Leave blank

1. Find $\int (6x^2 + 2 + x^{-\frac{1}{2}})\,dx$, giving each term in its simplest form.

(4)

$$\int (6x^2 + 2 + x^{-\frac{1}{2}})\,dx$$

$$= \frac{6x^3}{3} + 2x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2x^3 + 2x + 2x^{\frac{1}{2}} + c$$

Q1

(Total 4 marks)

Leave blank

5. Differentiate with respect to x

(a) $x^4 + 6\sqrt{x}$,

(3)

(b) $\dfrac{(x+4)^2}{x}$.

(4)

a) $y = x^4 + 6x^{\frac{1}{2}}$

$\dfrac{dy}{dx} = 4x^3 + \frac{1}{2} \times 6 \times x^{-\frac{1}{2}}$

$= 4x^3 + 3x^{-\frac{1}{2}}$

b) $y = \dfrac{(x+4)^2}{x} = \dfrac{x^2 + 8x + 16}{x}$

$= x + 8 + 16x^{-1}$

$\dfrac{dy}{dx} = 1 - 16x^{-2}$

May 2010

Leave blank

2. Find

$$\int (8x^3+6x^{\frac{1}{2}}-5)\,dx$$

giving each term in its simplest form.

(4)

$$\int (8x^3 + 6x^{\frac{1}{2}} - 5)\,dx$$

$$= \frac{8x^4}{4} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 5x + c$$

$$= 2x^4 + 4x^{\frac{3}{2}} - 5x + c$$

Q2

(Total 4 marks)

H 3 5 3 8 3 A 0 3 2 8

May 2010

Leave blank

7. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0$$

find $\frac{dy}{dx}$.

(6)

$y = 8x^3 - 4x^{\frac{1}{2}} + 3x + 2x^{-1}$

$\frac{dy}{dx} = 24x^2 - 2x^{-\frac{1}{2}} + 3 - 2x^{-2}$

May 2011

Leave blank

2. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(a) $\frac{dy}{dx}$, **(3)**

(b) $\int y \, dx$. **(4)**

$y = 2x^5 + 7 + x^{-3}$

a) $\frac{dy}{dx} = 10x^4 - 3x^{-4}$

$= 10x^4 - \frac{3}{x^4}$

b) $\int y \, dx$

$= \int (2x^5 + 7 + x^{-3}) \, dx$

$= \frac{2x^6}{6} + 7x + \frac{x^{-2}}{-2} + c$

$= \frac{1}{3}x^6 + 7x - \frac{1}{2x^2} + c$

C1 MAY 2012

Leave blank

1. Find

$$\int \left(6x^2 + \frac{2}{x^2} + 5\right) \mathrm{d}x$$

giving each term in its simplest form. $\int (6x^2 + 2x^{-2} + 5)\,dx$

(4)

$= \frac{6x^3}{3} + \frac{2x^{-1}}{-1} + 5x + c$

$= 2x^3 - 2x^{-1} + 5x + c$

Q1

(Total 4 marks)

P 4 0 6 8 4 A 0 2 2 4

CI MAY 2012

Leave blank

4.

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$$

(a) Find $\frac{dy}{dx}$ giving each term in its simplest form. **(4)**

(b) Find $\frac{d^2y}{dx^2}$ **(2)**

a) $\frac{dy}{dx} = 15x^2 - \frac{4\times6}{3}\times x^{\frac{1}{3}} + 2$

$= 15x^2 - 8x^{\frac{1}{3}} + 2$

b) $\frac{d^2y}{dx^2} = 30x - \frac{8}{3}x^{-\frac{2}{3}}$

C1 Jan 2013

Leave blank

11. The curve C has equation

$$y = 2x - 8\sqrt{x} + 5, \quad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving each term in its simplest form.

(3)

The point P on C has x-coordinate equal to $\frac{1}{4}$

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants.

(4)

The tangent to C at the point Q is parallel to the line with equation $2x - 3y + 18 = 0$

(c) Find the coordinates of Q.

(5)

a) $y = 2x - 8x^{\frac{1}{2}} + 5$

$\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}}$

b) $P \ (x = \frac{1}{4})$ gradient

$\frac{dy}{dx} = 2 - \frac{4}{\sqrt{x}} = 2 - \frac{4}{\sqrt{\frac{1}{4}}}$

$= 2 - \frac{4}{\frac{1}{2}} = 2 - 8 = -6$

y-coord at P

$y = 2 \times \frac{1}{4} - 8 \times \sqrt{\frac{1}{4}} + 5$

$y = \frac{1}{2} - 8 \times \frac{1}{2} + 5$

$y = \frac{1}{2} - 4 + 5 = \frac{3}{2}$ P is $(\frac{1}{4}, \frac{3}{2})$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y - \frac{3}{2} = -6(x - \frac{1}{4})$

$y - \frac{3}{2} = -6x + \frac{\cancel{6}^{3}}{\cancel{4}_{2}}$

$y = -6x + \frac{3}{2} + \frac{3}{2}$

$y = -6x + \frac{6}{2}$

$y = -6x + 3$

C1 Jan 2013

11c) $2x - 3y + 18 = 0$

$2x + 18 = 3y$

$\frac{2}{3}x + \frac{18}{3} = y$

gradient of tangent at Q is $\frac{2}{3}$

use $\frac{dy}{dx} = \frac{2}{3}$ to find x-coordinate

$\frac{2}{3} = 2 - \frac{4}{\sqrt{x}}$

$\frac{4}{\sqrt{x}} = 2 - \frac{2}{3}$

$\frac{4}{\sqrt{x}} = \frac{4}{3}$

So $\sqrt{x} = 3$

$x = 9$

Put $x = 9$ in equation for y

$y = 2 \times 9 - 8 \times \sqrt{9} + 5$

$y = 18 - 8 \times 3 + 5$

$y = 18 - 24 + 5$

$y = -1$

Coordinates of Q are $\underline{\underline{(9, -1)}}$

C1 MAY 2013

Leave blank

2. Find

$$\int \left(10x^4 - 4x - \frac{3}{\sqrt{x}}\right) dx$$

giving each term in its simplest form.

(4)

$\int (10x^4 - 4x - 3x^{-\frac{1}{2}})\, dx$

$= \frac{10x^5}{5} - \frac{4x^2}{2} - 3 \times 2x^{\frac{1}{2}} + c$

$= 2x^5 - 2x^2 - 6x^{\frac{1}{2}} + c$

Q2

(Total 4 marks)

P 4 1 8 0 2 A 0 3 2 8

Turn over

CI MAY 2013

Leave blank

9.

$$f'(x) = \frac{(3 - x^2)^2}{x^2}, \quad x \neq 0$$

(a) Show that $\quad f'(x) = 9x^{-2} + A + Bx^2,$

where A and B are constants to be found.

(3)

(b) Find $f''(x)$.

(2)

Given that the point $(-3, 10)$ lies on the curve with equation $y = f(x)$,

(c) find $f(x)$.

(5)

a) $f'(x) = \dfrac{(3-x^2)(3-x^2)}{x^2}$

$f'(x) = \dfrac{9 - 3x^2 - 3x^2 + x^4}{x^2} = \dfrac{9 - 6x^2 + x^4}{x^2}$

$f'(x) = 9x^{-2} - 6 + x^2$ in form required

where $\underline{\underline{A = -6}}$ and $\underline{\underline{B = 1}}$

b) $f''(x) = \underline{\underline{-18x^{-3} + 2x}}$ (differentiating)

c) Get $f(x)$ by integrating $f'(x)$

$f(x) = \dfrac{9}{-1}x^{-1} - 6x + \dfrac{x^3}{3} + c$

When $x = -3$, $f(x) = 10$

$10 = \dfrac{-9}{-3} - 6(-3) + \dfrac{(-3)^3}{3} + c$

$10 = +3 + 18 - 9 + c$

$10 - 3 - 18 + 9 = c$

$c = -2$

$\underline{\underline{f(x) = -9x^{-1} - 6x + \frac{1}{3}x^3 - 2}}$