Q6 (a) $y = 2^x$

$\ln y = \ln 2^x$

$\ln y = x \ln 2$

$y = e^{x \ln 2}$

$\Rightarrow 2^x = e^{x \ln 2}$

$$\frac{dy}{dx} = e^{x \ln 2} \times \ln 2$$

$$\frac{dy}{dx} = 2^x \times \ln 2$$

(b) $y = 2^{(x^2)}$

$$\frac{dy}{dx} = 2^{x^2} \ln 2 \times 2x$$

$$= 2x \times 2^{(x^2)} \ln 2$$

At $(2, 16)$ $= 2 \times 2 \times 2^4 \ln 2$

$$= 64 \ln 2$$

2. The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geq 0.$$

Use differentiation to find the value of $\frac{\mathrm{d}I}{\mathrm{d}t}$ when $t = 3$.
Give your answer in the form $\ln a$, where a is a constant.

(5)

$$\frac{dI}{dt} = -16(0.5)^t \ln(0.5)$$

$t = 3$ $\quad \frac{dI}{dt} = -16(0.5)^3 \ln(0.5)$

$$= -2 \ln 0.5$$

$$\frac{dI}{dt} = \ln 4$$

3. A curve C has equation

$$2^x + y^2 = 2xy.$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates $(3, 2)$.

(7)

3

$$2^x \ln 2 + 2y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$$

$$2^x \ln 2 - 2y = 2x\frac{dy}{dx} - 2y\frac{dy}{dx}$$

$$2^x \ln 2 - 2y = \frac{dy}{dx}(2x - 2y)$$

$$\frac{dy}{dx} = \frac{2^x \ln 2 - 2y}{2x - 2y}$$

At $(3, 2)$

$x = 3$
$y = 2$

$$\frac{dy}{dx} = \frac{2^3 \ln 2 - 4}{6 - 4}$$

$$= \frac{8 \ln 2 - 4}{2}$$

$$\frac{dy}{dx} = 4 \ln 2 - 2$$

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Leave blank

5.

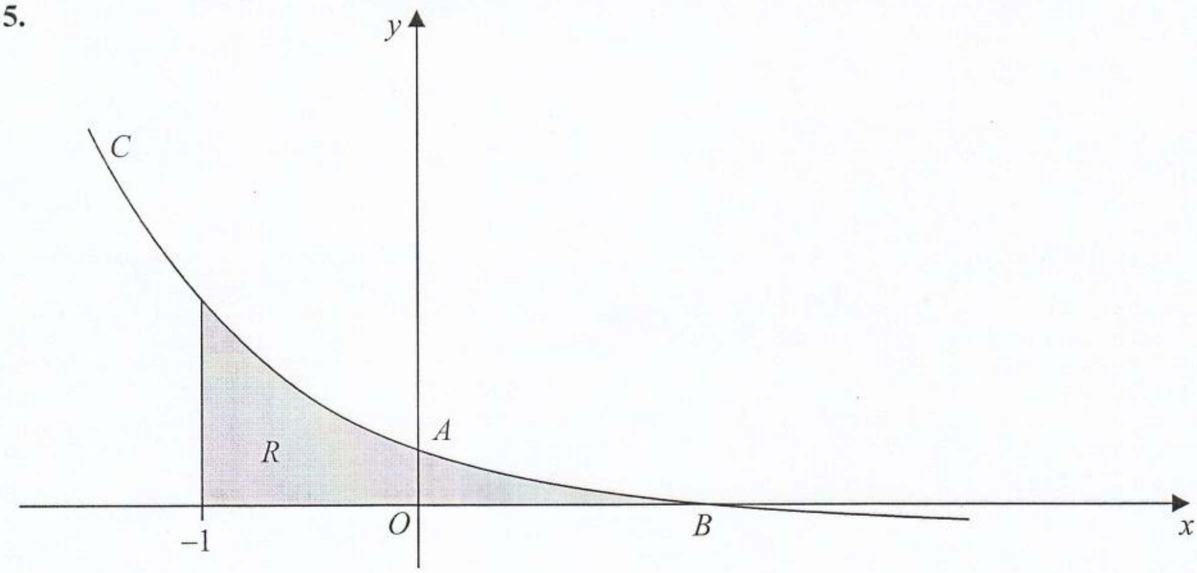

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3). **(2)**

(b) Find the x coordinate of the point B. **(2)**

(c) Find an equation of the normal to C at the point A. **(5)**

The region R, as shown shaded in Figure 2, is bounded by the curve C, the line $x = -1$ and the x-axis.

(d) Use integration to find the exact area of R. **(6)**

a) y-axis, $x = 0$ $\quad 0 = 1 - \frac{1}{2}t$

$t = 2$

Sub $t=2$ $\quad y = 2^2 - 1 = 3$, so A is (0,3)

b) at B, $y=0$, $0 = 2^t - 1$ so $t = 0$

sub $t = 0$, $x = 1 - \frac{1}{2} \times 0$, $x = 1$

x coordinate at B is $\underline{\underline{x = 1}}$

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$x = 1 - \frac{1}{2}t$ $\qquad$ $y = 2^t - 1$

5.c) $\frac{dx}{dt} = -\frac{1}{2}$ $\qquad$ $\frac{dy}{dt} = 2^t \ln 2$

learn this result

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\boxed{\begin{array}{l} \frac{d}{dt}(2^t) = 2^t \ln 2 \\ \frac{d}{dt}(3^t) = 3^t \ln 3 \end{array}}$

$\frac{dy}{dx} = \frac{2^t \ln 2}{-\frac{1}{2}} = -2^t \times 2 \ln 2$

at A, $t = 2$

gradient at A, $\frac{dy}{dx} = (-2)^2 \times 2 \ln 2$

$= 8 \ln 2$

Gradient of normal at A is $\frac{-1}{8 \ln 2}$
(perpendicular)

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{8 \ln 2}(x - 0)$

$\underline{y = -\frac{1}{8 \ln 2}x + 3}$ is equation of normal at C

5d) at $x = 1$, $1 = 1 - \frac{1}{2}t$, $t = 0$ (limits)

at $x = -1$, $-1 = 1 - \frac{1}{2}t$, $-2 = -\frac{1}{2}t$, $t = 4$

$\int_{-1}^{1} y \, dx = \int_{4}^{0} y \frac{dx}{dt} \, dt = \int_{4}^{0} (2^t - 1) \times -\frac{1}{2} \, dt$

$= \frac{1}{2} \int_{0}^{4} (2^t - 1) \, dt$

switch limits getting rid of minus sign

$= \frac{1}{2} \left[\frac{2^t}{\ln 2} - t \right]_0^4$

$= \frac{1}{2} \left[\left(\frac{16}{\ln 2} - 4 \right) - \left(\frac{1}{\ln 2} - 0 \right) \right]$

$= \frac{1}{2} \left(\frac{16}{\ln 2} - \frac{1}{\ln 2} - 4 \right)$

$= \frac{1}{2} \left(\frac{15}{\ln 2} - 4 \right)$

$= \underline{\frac{15}{2 \ln 2} - 2}$

If $y = 2^t$

$\frac{dy}{dt} = 2^t \ln 2$

$\therefore$

$\int 2^t \, dt = \frac{2^t}{\ln 2} + c$