6.

Figure 1

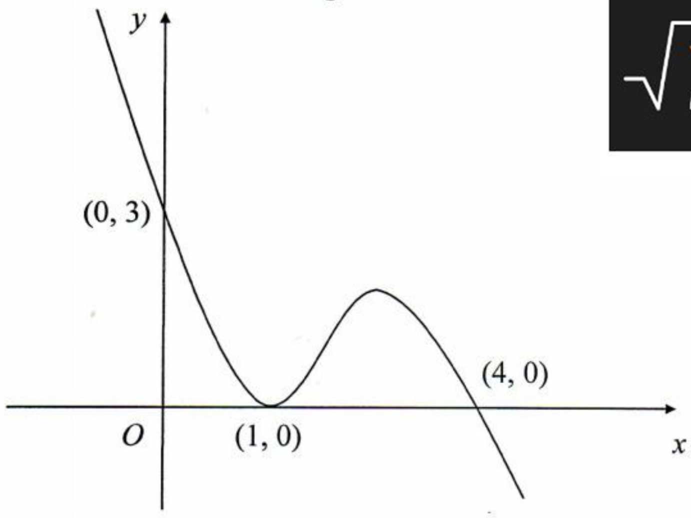


Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve passes through the points (0, 3) and (4, 0) and touches the x-axis at the point (1, 0).

On separate diagrams, sketch the curve with equation

(*a*) $y = f(x + 1)$, shifted one unit left **(3)**

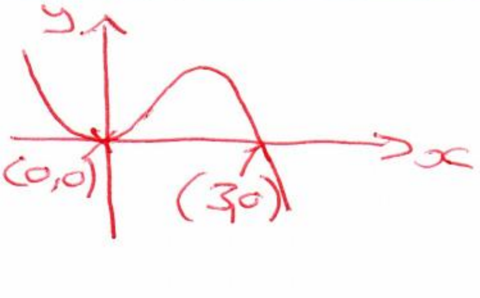
(*b*) $y = 2f(x)$, Twice as high **(3)**

(*c*) $y = f\left(\frac{1}{2}x\right)$. Stretch ×2 parallel to x-axis **(3)**

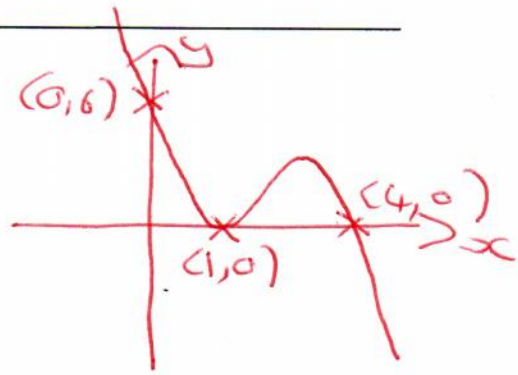
On each diagram show clearly the coordinates of all the points at which the curve meets the axes.

---

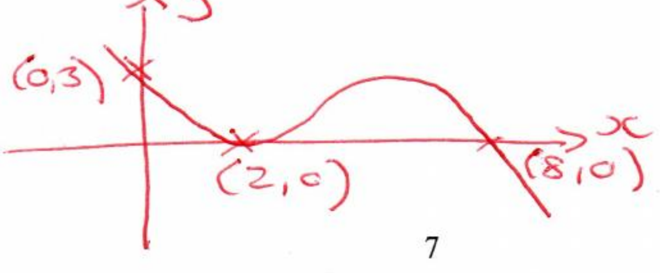
a)



b)



c)



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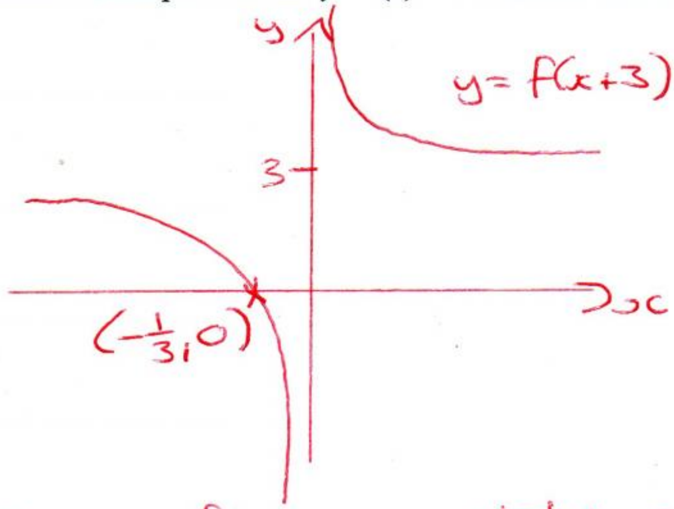
3. Given that $$f(x) = \frac{1}{x}, \quad x \neq 0,$$

(a) sketch the graph of $y = f(x) + 3$ and state the equations of the asymptotes.

(4)

(b) Find the coordinates of the point where $y = f(x) + 3$ crosses a coordinate axis.

(2)



a) equation of assymptotes are

$x = 0$ (y-axis)

$y = 3$

b) Crosses x-axis ($y=0$)

$0 = \frac{1}{x} + 3$

$\frac{1}{x} = -3$

$\therefore x = -\frac{1}{3}$

Coordinates are $(-\frac{1}{3}, 0)$ where curve meets the x-axis

10. (a) On the same axes sketch the graphs of the curves with equations

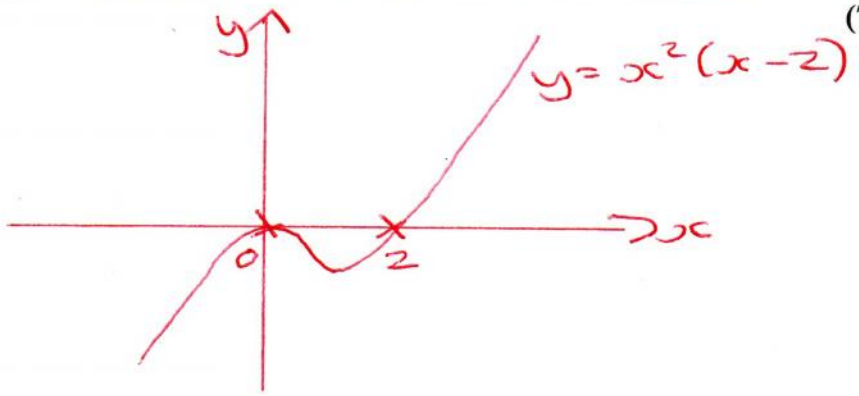
(i) $y = x^2(x-2)$, **(3)**

(ii) $y = x(6-x)$, **(3)**

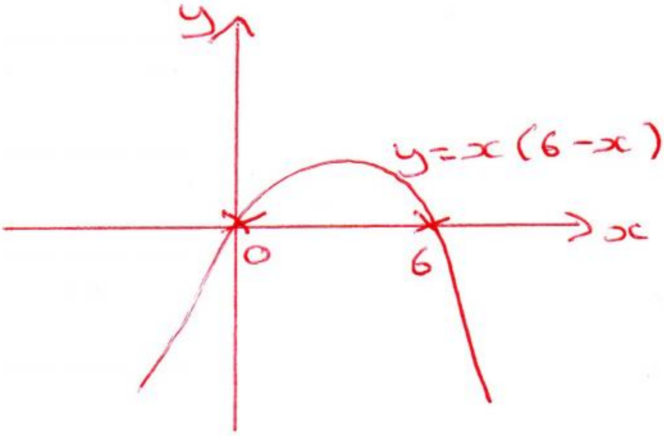
and indicate on your sketches the coordinates of all the points where the curves cross the x-axis.

(b) Use algebra to find the coordinates of the points where the graphs intersect. **(7)**

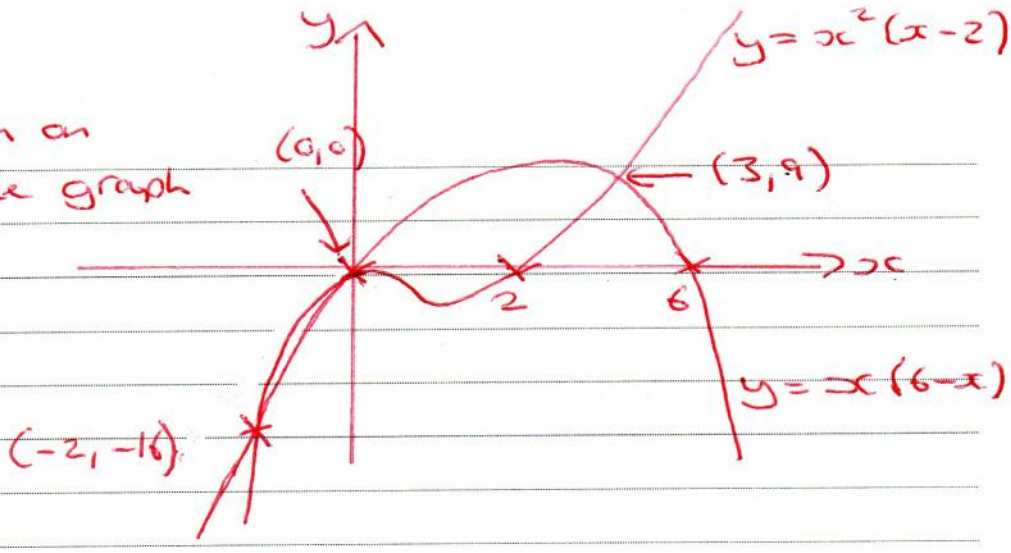
a) (i)



(ii)



Both on same graph



10b) $y = x^2(x-2)$ ①

$y = x(6-x)$ ②

To find where graphs intersect, solve simultaneously

Put equation ① = ②

$\therefore x^2(x-2) = x(6-x)$

$x^3 - 2x^2 = 6x - x^2$

$x^3 - 2x^2 + x^2 - 6x = 0$

$x^3 - x^2 - 6x = 0$

$x(x^2 - x - 6) = 0$

$x(x+2)(x-3) = 0$

Either $x = 0$ or $x = -2$ or $x = 3$

when $x = 0$, in ②, $y = 0$

$x = -2$, in ②, $y = -2(6--2) = -16$

$x = 3$, in ②, $y = 3(6-3) = 9$

Coordinates of intersection are

$(0,0)$, $(-2,-16)$, $(3,9)$

6.

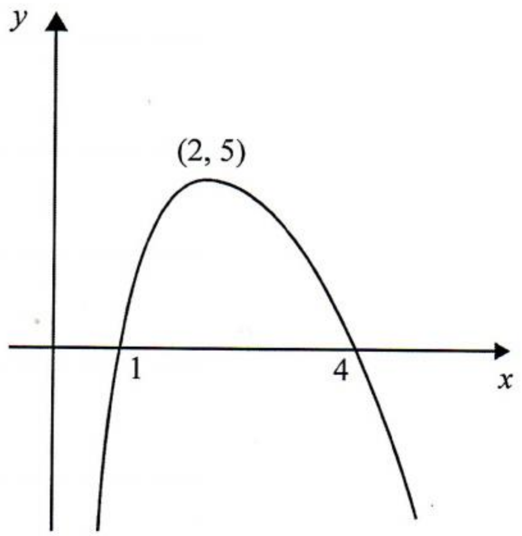


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve crosses the x-axis at the points $(1, 0)$ and $(4, 0)$. The maximum point on the curve is $(2, 5)$.
In separate diagrams sketch the curves with the following equations.
On each diagram show clearly the coordinates of the maximum point and of each point at which the curve crosses the x-axis.

(a) $y = 2\mathrm{f}(x)$,

(3)

(b) $y = \mathrm{f}(-x)$.

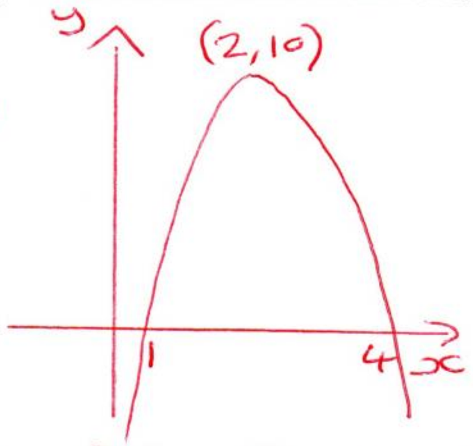
(3)

The maximum point on the curve with equation $y = \mathrm{f}(x + a)$ is on the y-axis.

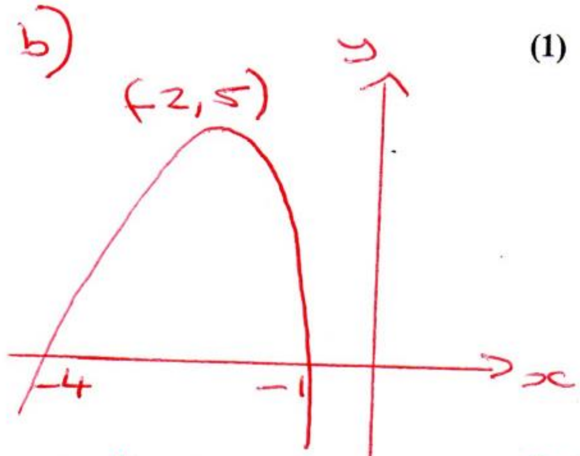
(c) Write down the value of the constant a.

(1)

a)



b)



c) $f(x+2)$ would shift the graph $y=f(x)$ 2 units to the left $\therefore a=2$

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5.

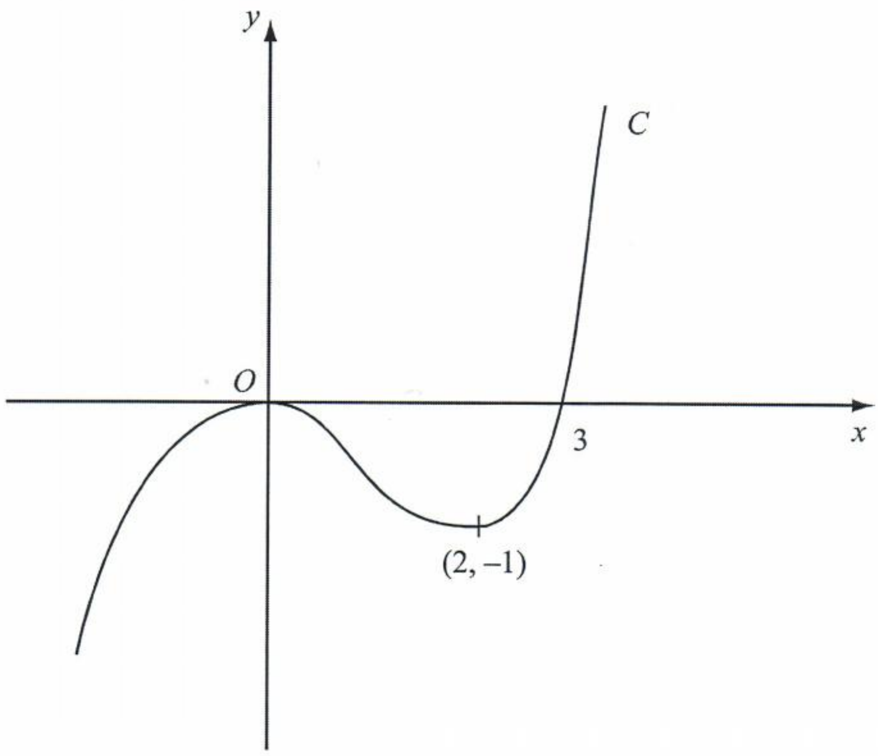


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$. There is a maximum at $(0, 0)$, a minimum at $(2, -1)$ and C passes through $(3, 0)$.

On separate diagrams sketch the curve with equation

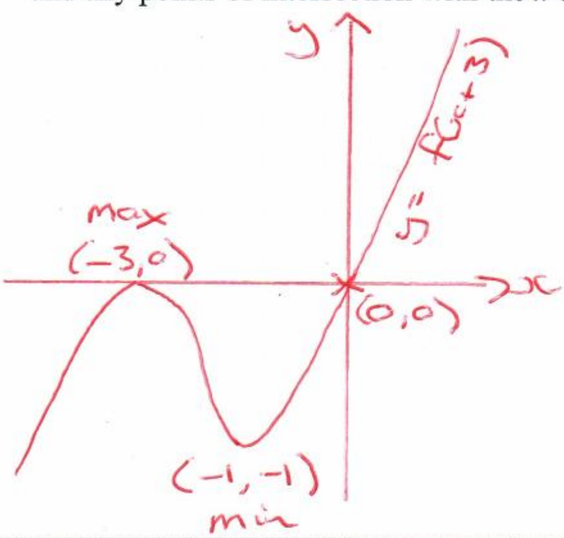
(a) $y = \mathrm{f}(x + 3)$,

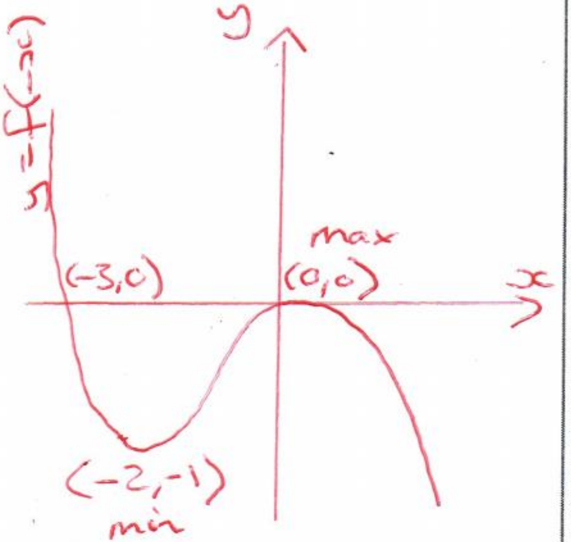
(3)

(b) $y = \mathrm{f}(-x)$.

(3)

On each diagram show clearly the coordinates of the maximum point, the minimum point and any points of intersection with the x-axis.





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8. The point P (1, a) lies on the curve with equation $y = (x + 1)^2(2 - x)$.

(a) Find the value of a. **(1)**

when $x = 1$,
$y = (1+1)^2(2-1) = 4$
$\therefore a = 4$

(b) On the axes below sketch the curves with the following equations:

(i) $y = (x + 1)^2(2 - x)$, when $x = 0$, $y = 2$

(ii) $y = \dfrac{2}{x}$.

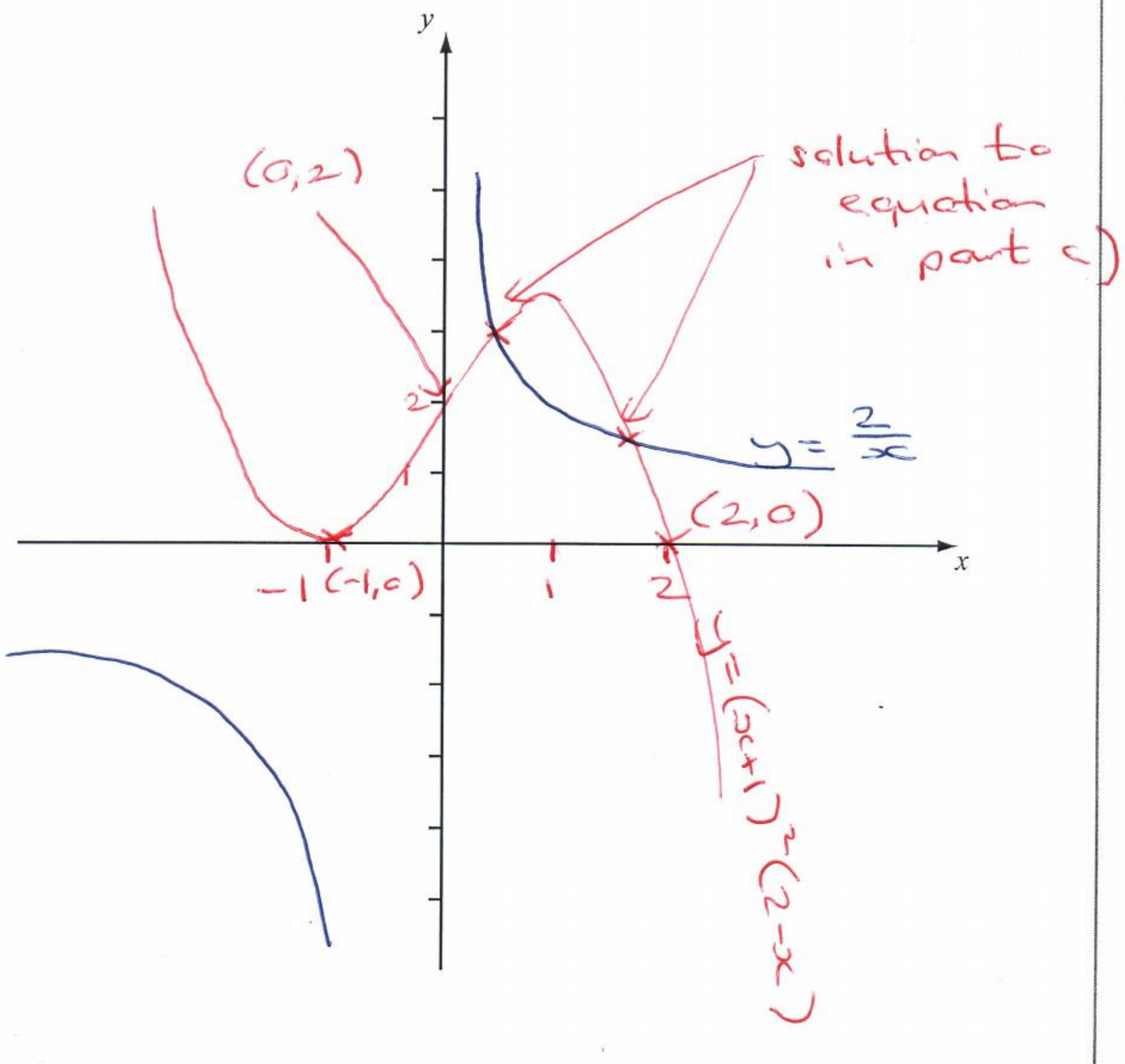
On your diagram show clearly the coordinates of any points at which the curves meet the axes. **(5)**

(c) With reference to your diagram in part (b) state the number of real solutions to the equation

$$(x+1)^2(2-x) = \frac{2}{x}.$$

(1)

2 intersections therefore 2 roots



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5.

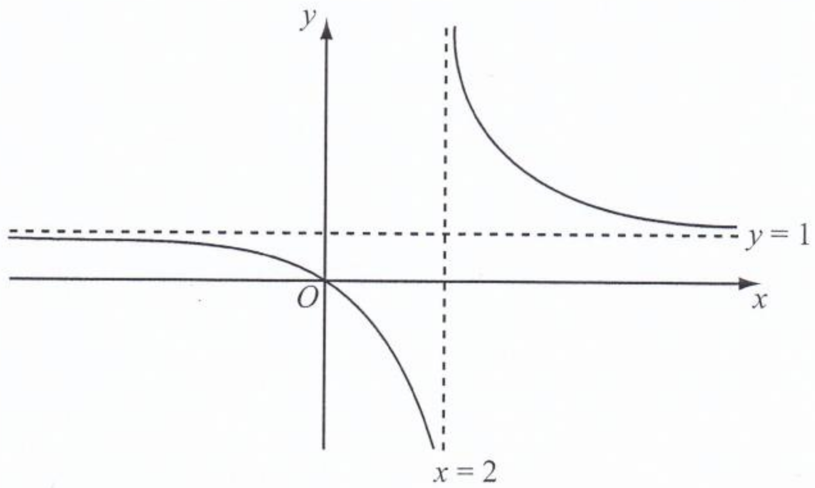


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{x}{x-2}, \qquad x \neq 2$$

The curve passes through the origin and has two asymptotes, with equations $y = 1$ and $x = 2$, as shown in Figure 1.

— shifted 1 unit to right

(a) In the space below, sketch the curve with equation $y = \mathrm{f}(x-1)$ and state the equations of the asymptotes of this curve.

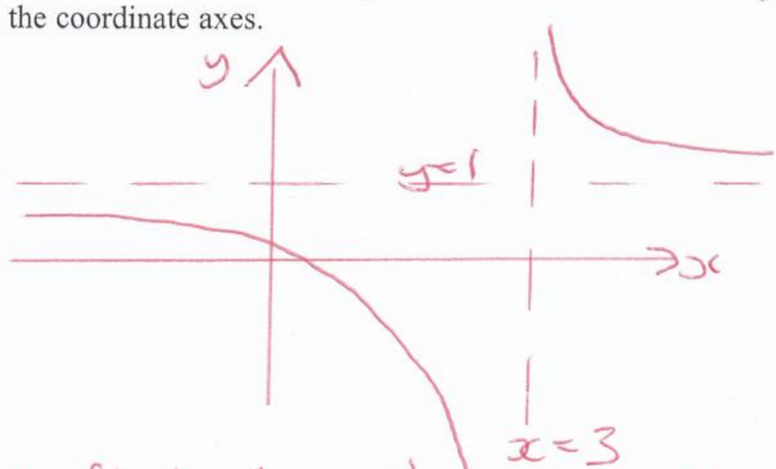
→ $y=1$, $x=3$ are equation of assymptotes

(3)

(b) Find the coordinates of the points where the curve with equation $y = \mathrm{f}(x-1)$ crosses the coordinate axes.

(4)

a)



Shifted 1 unit across, so crosses x-axis at $(1,0)$

b) $f(x) = \dfrac{x}{x-2}$ $\therefore f(x-1) = \dfrac{(x-1)}{(x-1)-2} = \dfrac{x-1}{x-3}$

Crosses y-axis when $x=0$

$\therefore y = \dfrac{0-1}{0-3} = \dfrac{1}{3}$ $\quad (0, \frac{1}{3})$

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10. (a) On the axes below, sketch the graphs of

(i) $y = x(x+2)(3-x)$

cubic which meets x-axis at $x=0$, $x=-2$, $x=3$

(ii) $y = -\dfrac{2}{x}$

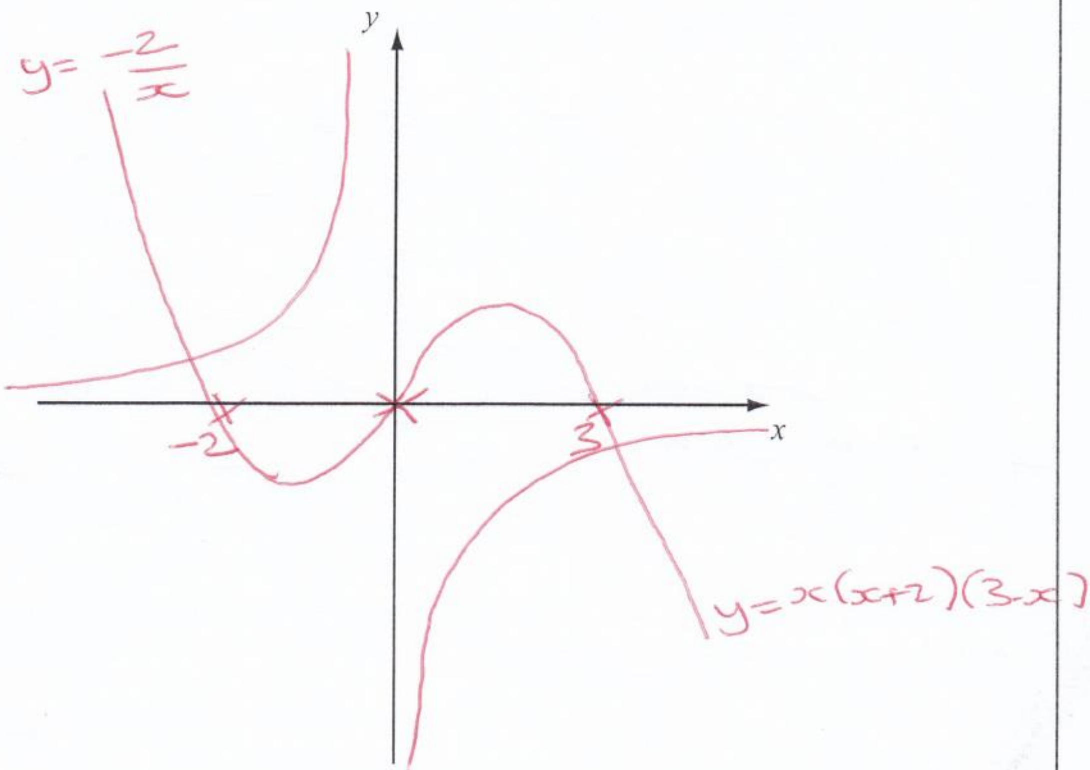
showing clearly the coordinates of all the points where the curves cross the coordinate axes.

(6)

(b) Using your sketch state, giving a reason, the number of real solutions to the equation

$$x(x+2)(3-x) + \frac{2}{x} = 0$$

(2)



b) 2 solution as there are 2 intersection points

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8. The curve C_1 has equation

$$y = x^2(x+2)$$

(a) Find $\frac{dy}{dx}$ **(2)**

(b) Sketch C_1, showing the coordinates of the points where C_1 meets the x-axis. **(3)**

(c) Find the gradient of C_1 at each point where C_1 meets the x-axis. **(2)**

The curve C_2 has equation

$$y = (x-k)^2(x-k+2)$$

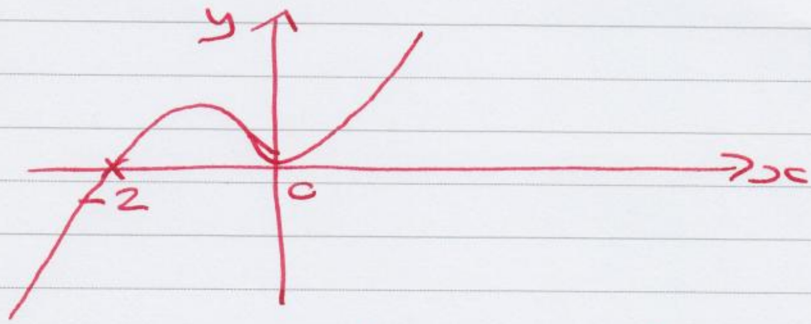
where k is a constant and $k > 2$

(d) Sketch C_2, showing the coordinates of the points where C_2 meets the x and y axes. **(3)**

a) $y = x^3 + 2x^2$

$\frac{dy}{dx} = 3x^2 + 4x$

b)

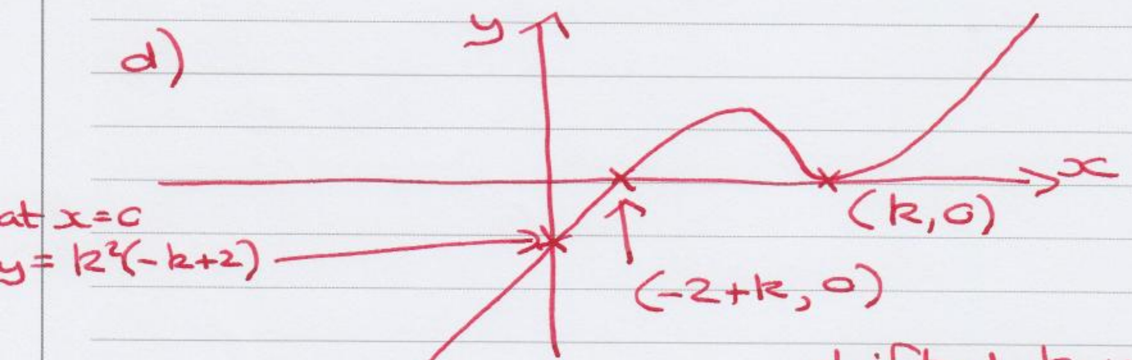


c) at $x = -2$, $\frac{dy}{dx} = 3(-2)^2 + 4(-2)$

$= 12 - 8 = 4$

at $x = 0$, $\frac{dy}{dx} = 0$

d)



shifted k units right

5.

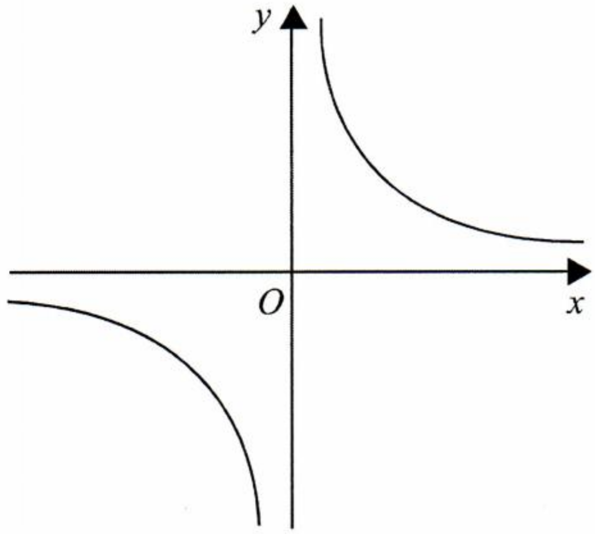


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{3}{x}$, $x \neq 0$.

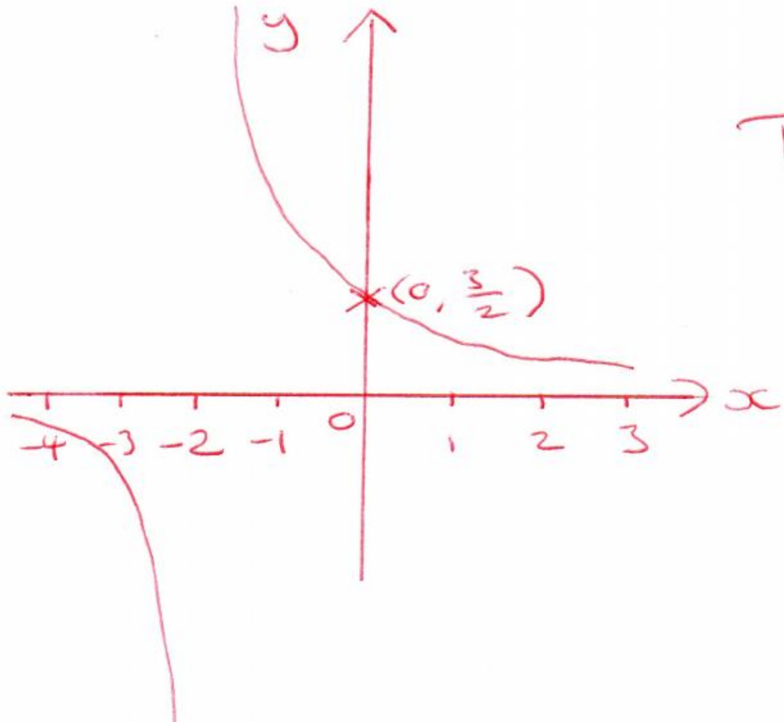
(a) On a separate diagram, sketch the curve with equation $y = \frac{3}{x+2}$, $x \neq -2$,

showing the coordinates of any point at which the curve crosses a coordinate axis.

(3)

(b) Write down the equations of the asymptotes of the curve in part (a).

(2)



Translation of curve 2 units to the left

crosses y-axis

$y = \frac{3}{0+2} = \frac{3}{2}$

b) Assymptotes are $x = -2$ and $y = 0$

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3.

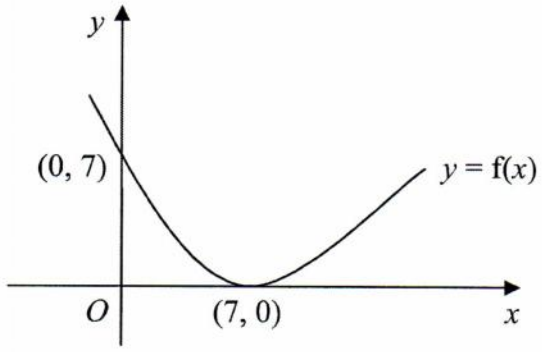


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve passes through the point (0, 7) and has a minimum point at (7, 0).

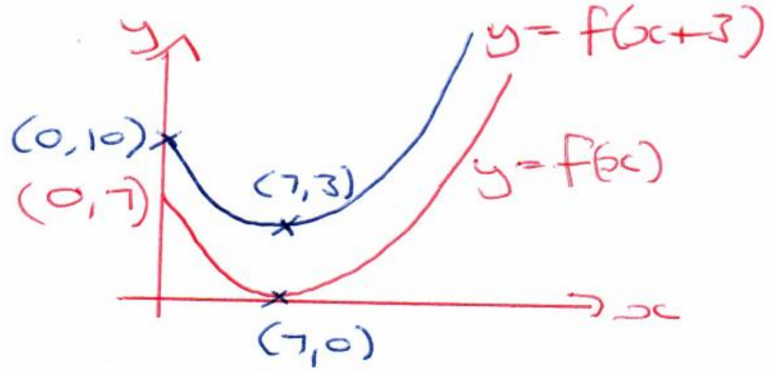
On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(x) + 3$, ← raises by 3 units
(3)

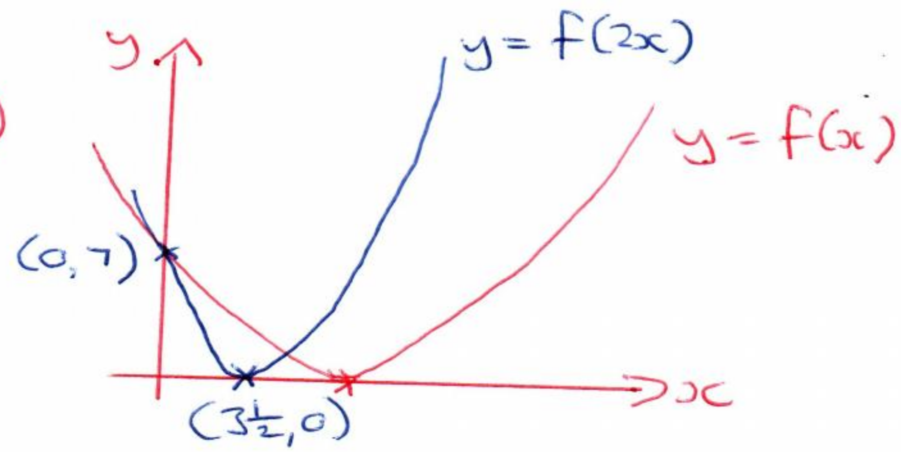
(b) $y = \mathrm{f}(2x)$. ← stretch by scale factor $\frac{1}{2}$ graph squeezed towards y-axis
(2)

On each diagram, show clearly the coordinates of the minimum point and the coordinates of the point at which the curve crosses the y-axis.

a)



b)



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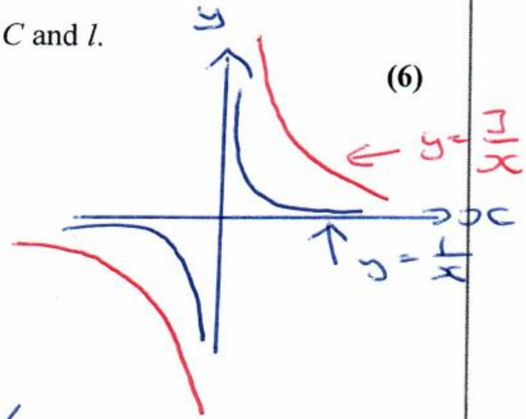
6. The curve C has equation $y = \frac{3}{x}$ and the line l has equation $y = 2x + 5$.

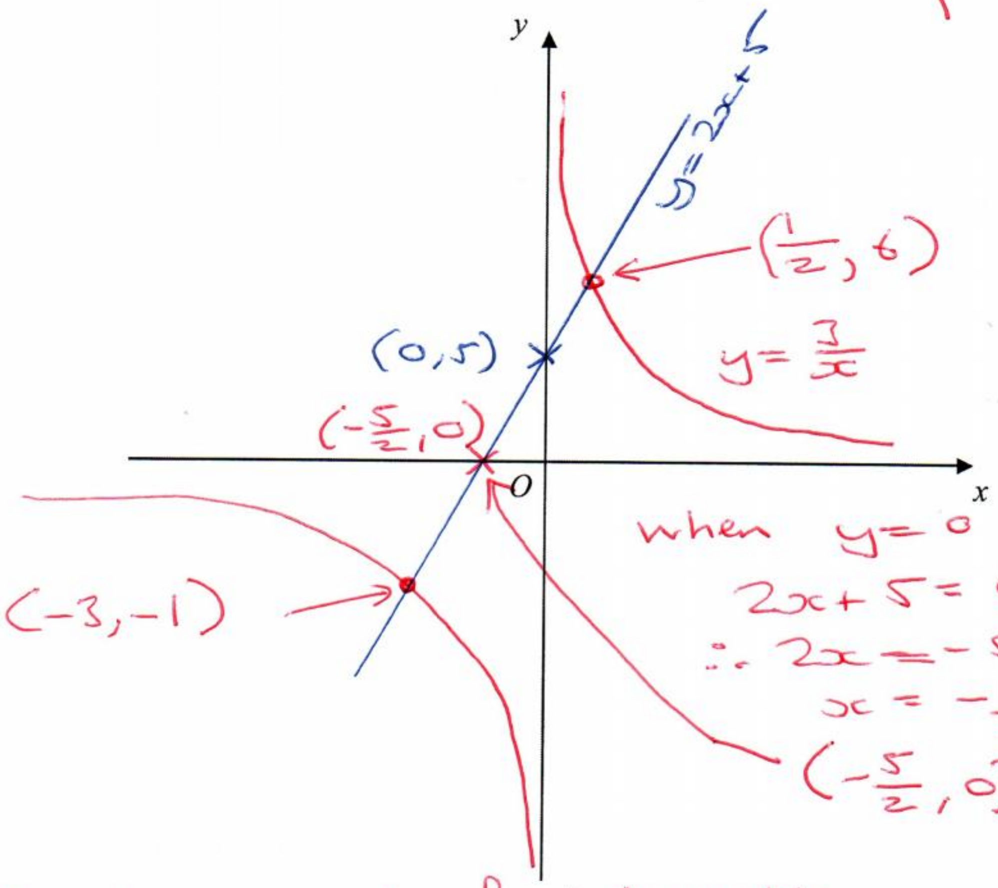
(a) On the axes below, sketch the graphs of C and l, indicating clearly the coordinates of any intersections with the axes.

(3)

(b) Find the coordinates of the points of intersection of C and l.

(6)





when $y = 0$

$2x + 5 = 0$

$\therefore 2x = -5$

$x = -\frac{5}{2}$

$(-\frac{5}{2}, 0)$

b) At point of intersection

$2x + 5 = \frac{3}{x}$ (x through by x)

$2x^2 + 5x = 3$

$2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

$\therefore 2x - 1 = 0$ or $x + 3 = 0$

$x = \frac{1}{2}$ or $x = -3$

when $x = \frac{1}{2}$, $y = \frac{3}{\frac{1}{2}} = 6$

when $x = -3$, $y = \frac{3}{-3} = -1$

$\therefore$ Points of intersection $(\frac{1}{2}, 6)$, $(-3, -1)$

10. (a) Factorise completely $x^3 - 6x^2 + 9x$

(3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.

(4)

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

$$y = (x-2)^3 - 6(x-2)^2 + 9(x-2)$$

showing the coordinates of the points at which the curve meets the x-axis.

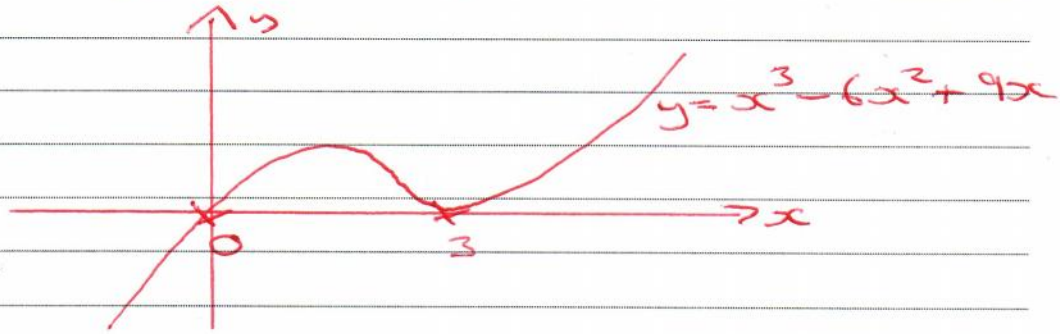
(2)

a) $x^3 - 6x^2 + 9x$

$= x(x^2 - 6x + 9)$

$= x(x-3)(x-3)$

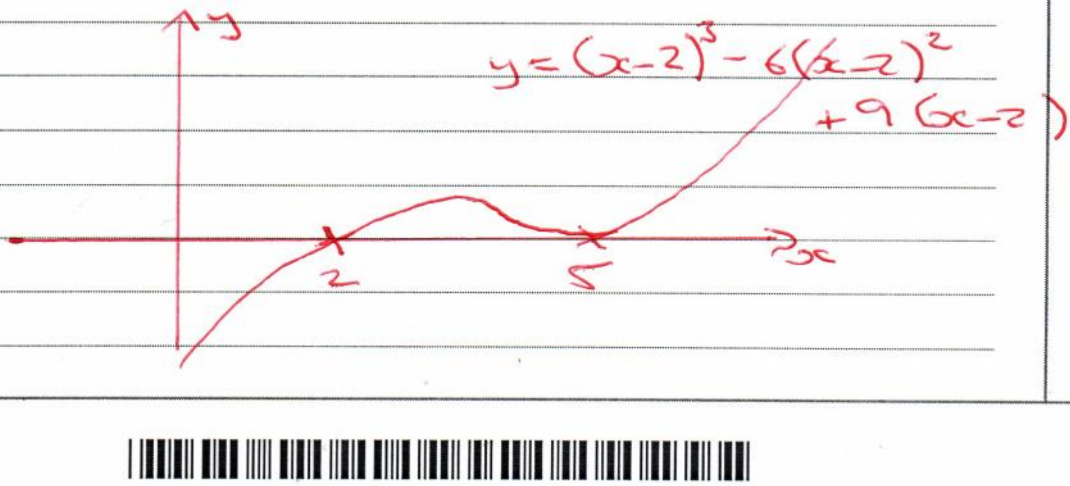
b)



c) $y = (x-2)^3 - 6(x-2)^2 + 9(x-2)$

is $y = x^3 - 6x^2 + 9x$

shifted 2 units to the right



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3. On separate diagrams, sketch the graphs of

(a) $y = (x + 3)^2$,

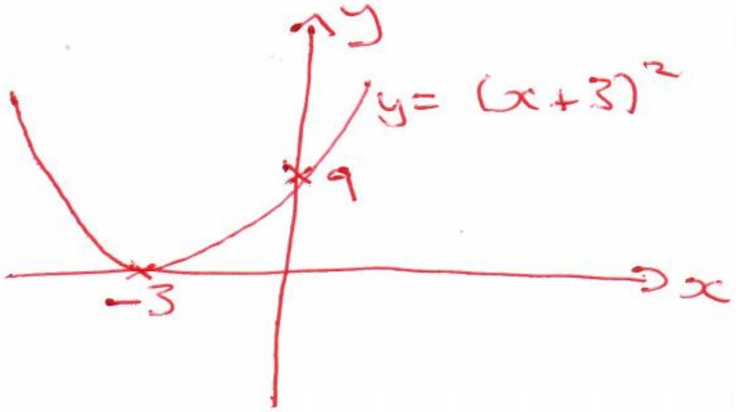
(3)

(b) $y = (x + 3)^2 + k$, where k is a positive constant.

(2)

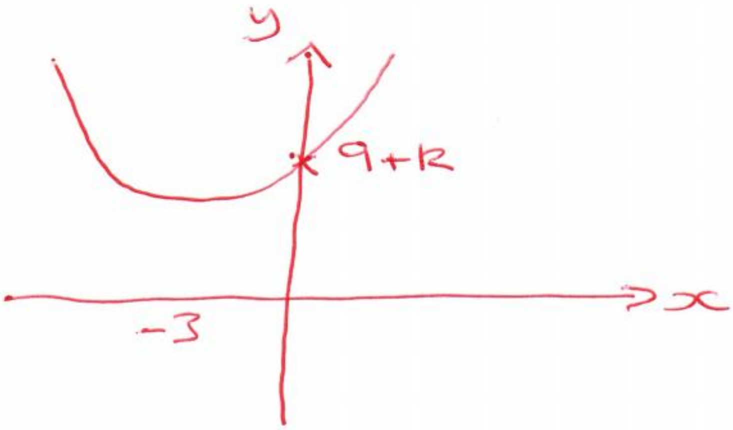
Show on each sketch the coordinates of each point at which the graph meets the axes.

a)



Meets axes at $(-3,0)$ and $(0,9)$

b)



Meets y-axis at $(0, 9+k)$

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6.

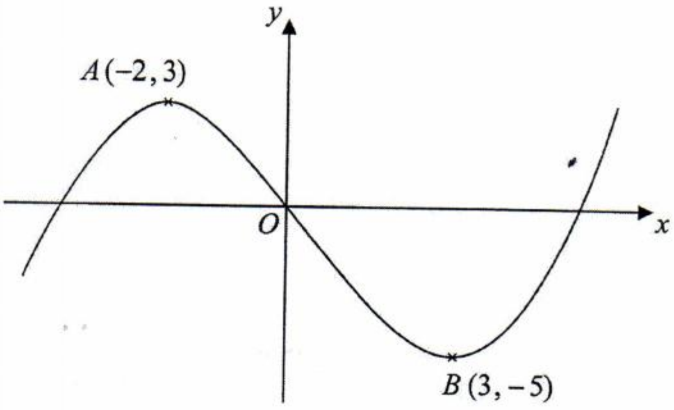


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$. The curve has a maximum point A at $(-2, 3)$ and a minimum point B at $(3, -5)$.

On separate diagrams sketch the curve with equation

(a) $y = \mathrm{f}(x+3)$

(3)

(b) $y = 2\mathrm{f}(x)$

(3)

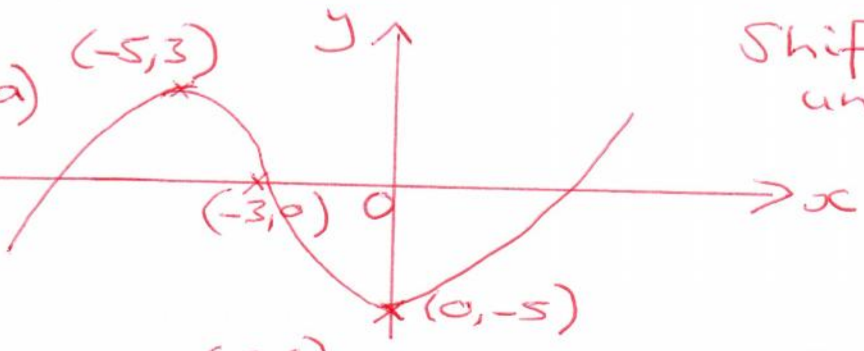
On each diagram show clearly the coordinates of the maximum and minimum points.

The graph of $y = \mathrm{f}(x) + a$ has a minimum at $(3, 0)$, where a is a constant.

(c) Write down the value of a.

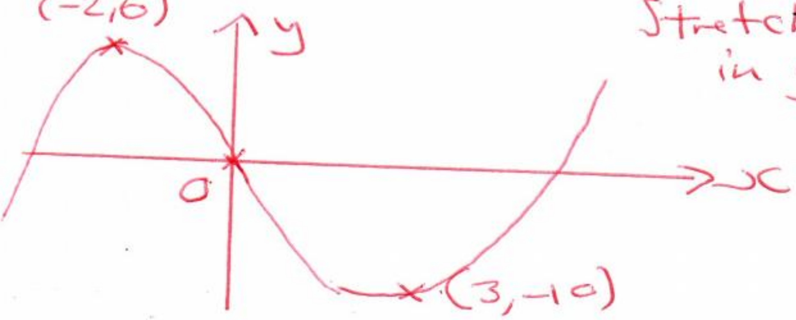
(1)

a)



Shifted 3 units left

b)



Stretched x2 in y direction

c) $a = 5$ (graph shifted up 5 units)

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10. (a) On the axes below sketch the graphs of

(i) $y = x(4-x)$ $y = -x^2$ graph

(ii) $y = x^2(7-x)$ $y = -x^3$ graph

showing clearly the coordinates of the points where the curves cross the coordinate axes.

(5)

(b) Show that the x-coordinates of the points of intersection of

$$y = x(4-x) \quad \text{and} \quad y = x^2(7-x)$$

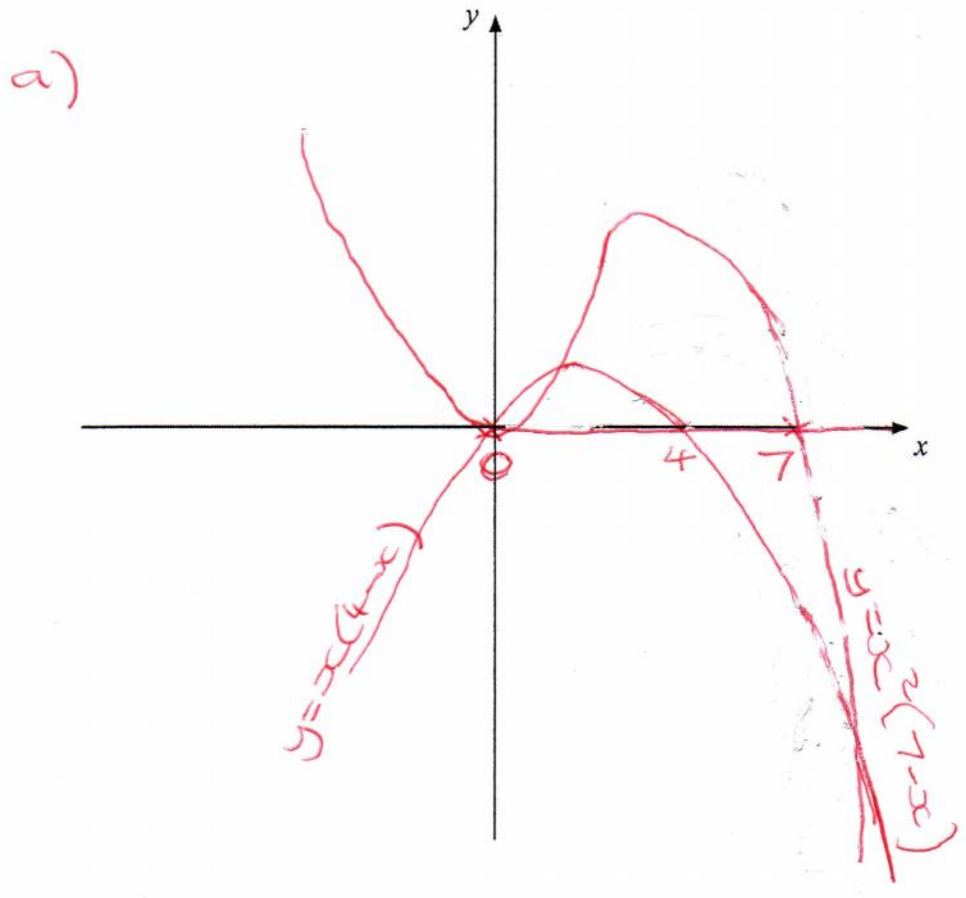
are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$

(3)

The point A lies on both of the curves and the x and y coordinates of A are both positive.

(c) Find the exact coordinates of A, leaving your answer in the form $(p + q\sqrt{3},\ r + s\sqrt{3})$, where p, q, r and s are integers.

(7)



H 3 5 3 8 3 A 0 2 2 2 8

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10b) $y = x(4-x)$ ①
$y = x^2(7-x)$ ②

For intersection, equate the above equations and solve

let ① = ②

$x(4-x) = x^2(7-x)$

$4x - x^2 = 7x^2 - x^3$

$x^3 - 7x^2 - x^2 + 4x = 0$

$x^3 - 8x^2 + 4x = 0$

$x(x^2 - 8x + 4) = 0$

as required

c) solve $x(x^2 - 8x + 4) = 0$

Either $x = 0$ or $x^2 - 8x + 4 = 0$

Completing the square

$x^2 - 8x + 4 = 0$

$(x-4)^2 - 16 + 4 = 0$

$(x-4)^2 - 12 = 0$

$(x-4)^2 = 12$

$x - 4 = \pm\sqrt{12}$

So $x = 4 + \sqrt{12}$ or $x = 4 - \sqrt{12}$

using ①, with $x = 4+\sqrt{12}$
$y = (4+\sqrt{12})(4-(4+\sqrt{12}))$
$y = (4+\sqrt{12})(-\sqrt{12})$
$y = -4\sqrt{12} - 12$ (not positive value

using ① with $x = 4-\sqrt{12}$
$y = (4-\sqrt{12})(4-(4-\sqrt{12}))$
$y = (4-\sqrt{12})(\sqrt{12})$
$y = 4\sqrt{12} - 12$

coordinates of A are
$(4-\sqrt{12}, 4\sqrt{12} - 12)$

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10 c) continued

Question asks for form

$(p + q\sqrt{3}, r + s\sqrt{3})$

$$(4 - \sqrt{4}\sqrt{3}, 4\sqrt{4}\sqrt{3} - 12)$$

$$= (4 - 2\sqrt{3}, -12 + 8\sqrt{3})$$

where $p = 4$

$q = -2$

$r = -12$

$s = 8$

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8.

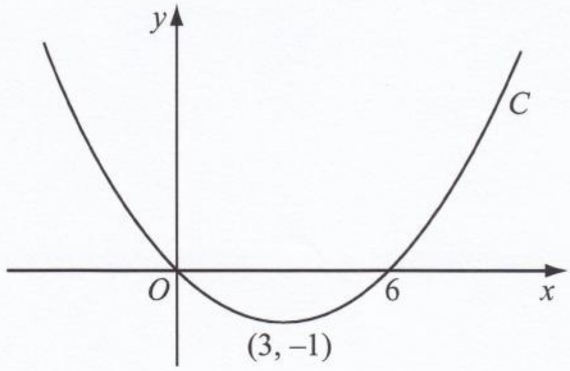


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.
The curve C passes through the origin and through $(6, 0)$.
The curve C has a minimum at the point $(3, -1)$.

On separate diagrams, sketch the curve with equation

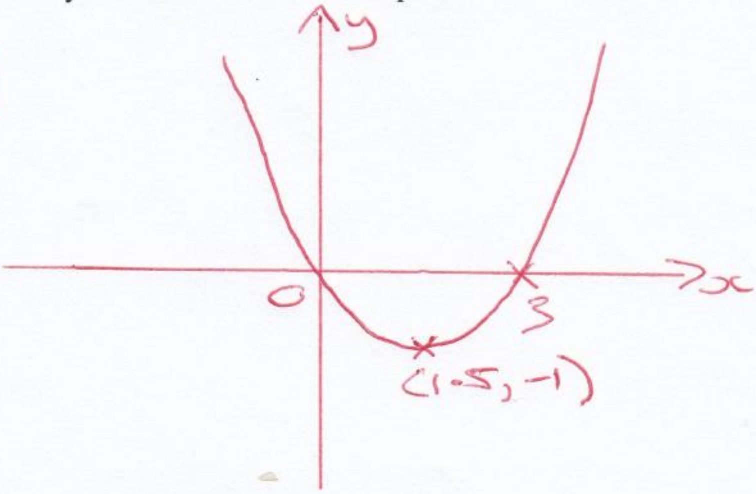
(a) $y = \mathrm{f}(2x)$, divide x - coords by 2 **(3)**

(b) $y = -\mathrm{f}(x)$, reflect in x - axis **(3)**

(c) $y = \mathrm{f}(x + p)$, where p is a constant and $0 < p < 3$.
shifted p units to the left **(4)**

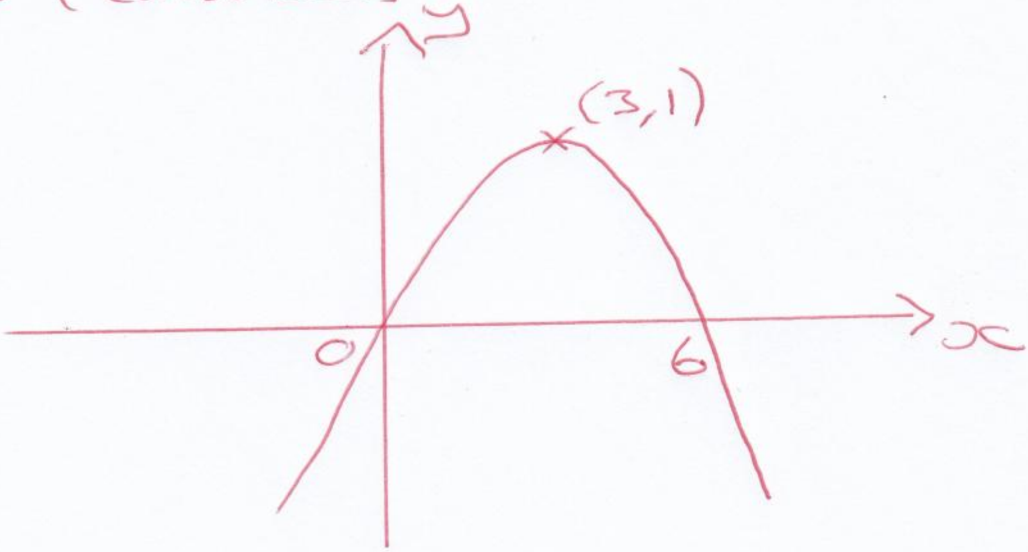
On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

a)

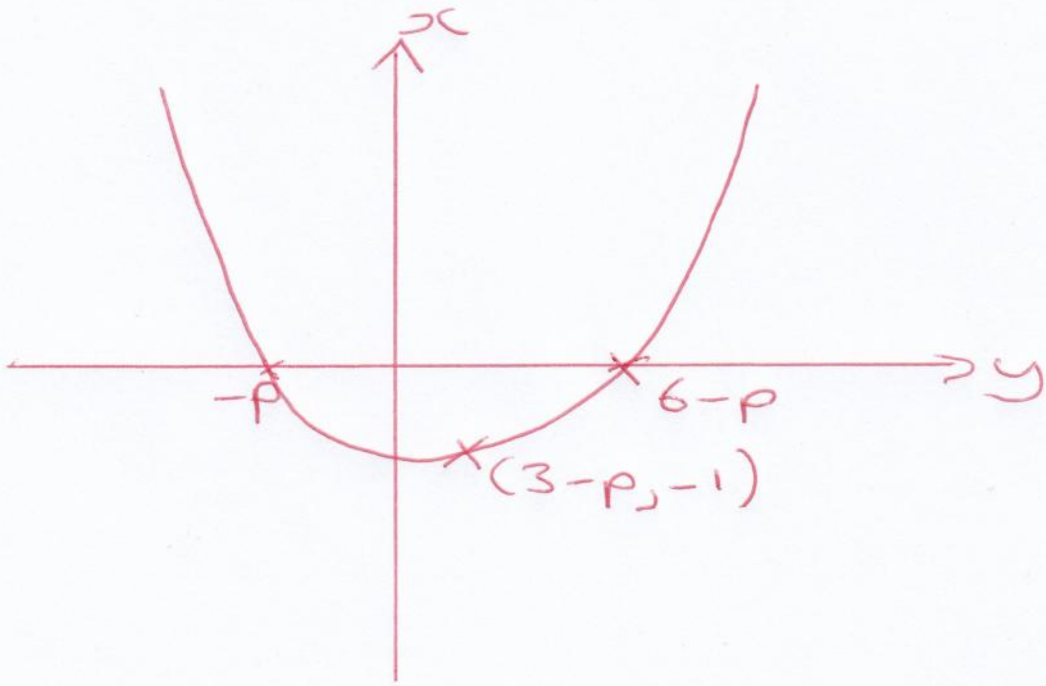


May 2011

Q8 (continued)



Q9



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10.

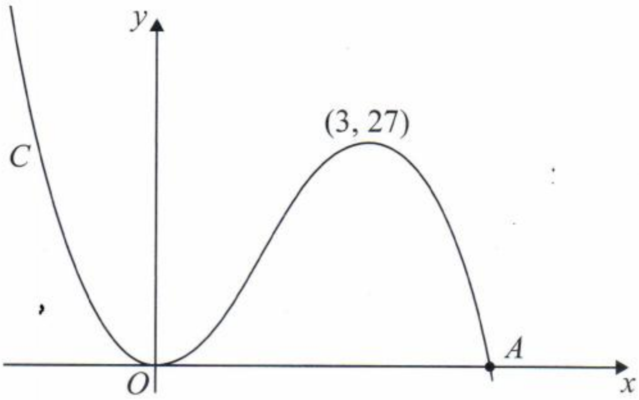


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^2(9 - 2x)$$

There is a minimum at the origin, a maximum at the point (3, 27) and C cuts the x-axis at the point A.

(a) Write down the coordinates of the point A.

(1)

(b) On separate diagrams sketch the curve with equation

(i) $y = \mathrm{f}(x + 3)$

(ii) $y = \mathrm{f}(3x)$

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes.

(6)

The curve with equation $y = \mathrm{f}(x) + k$, where k is a constant, has a maximum point at (3, 10).

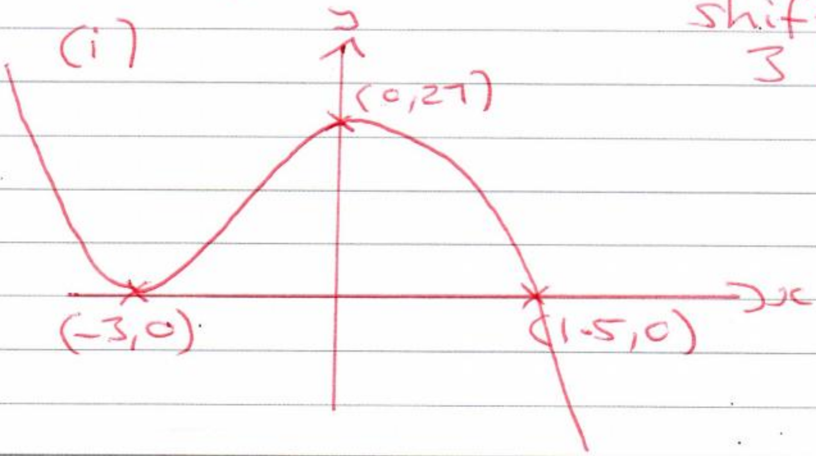
(c) Write down the value of k.

(1)

a) $A(4.5, 0)$

$y = f(x+3)$ shifted 3 units left

b) (i)



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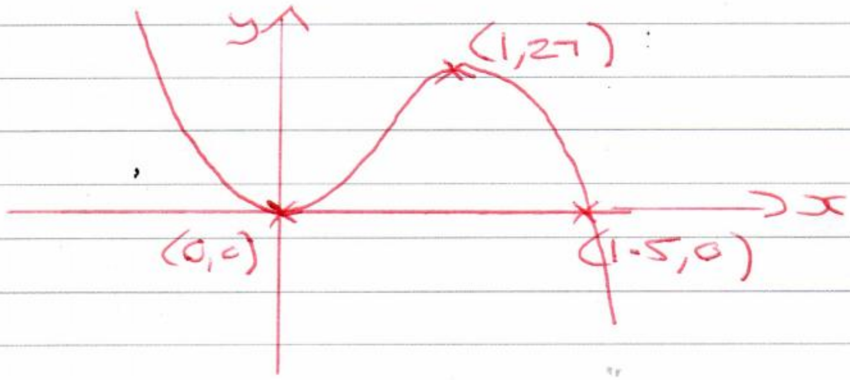
Question 10 continued

(ii) $y = f(3x)$ $\Rightarrow \leftarrow \div x$ by 3



c) If maximum pt is (3,10)
it has gone down 17 units

$y = f(x) + k$

$k = -17$

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6.

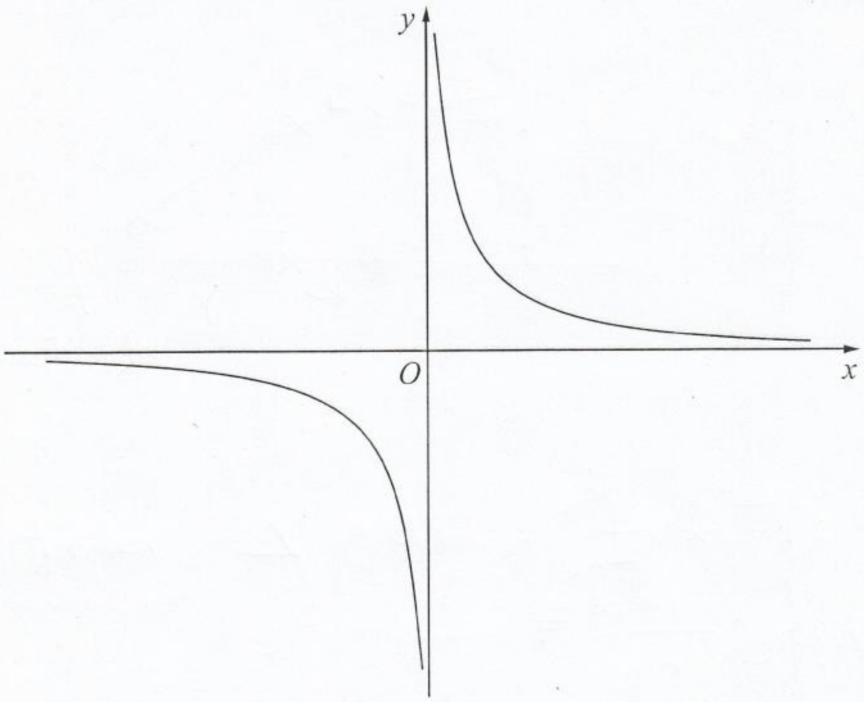


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{2}{x}, \ x \neq 0$

shifted down 5 units

The curve C has equation $y = \frac{2}{x} - 5, \ x \neq 0$, and the line l has equation $y = 4x + 2$

$y = 0, \ 5 = \frac{2}{x}$ so $x = \frac{2}{5}$ $\quad x = 0, y = 2$

$y = 0, x = -\frac{1}{2}$

(a) Sketch and clearly label the graphs of C and l on a single diagram.

On your diagram, show clearly the coordinates of the points where C and l cross the coordinate axes.

(5)

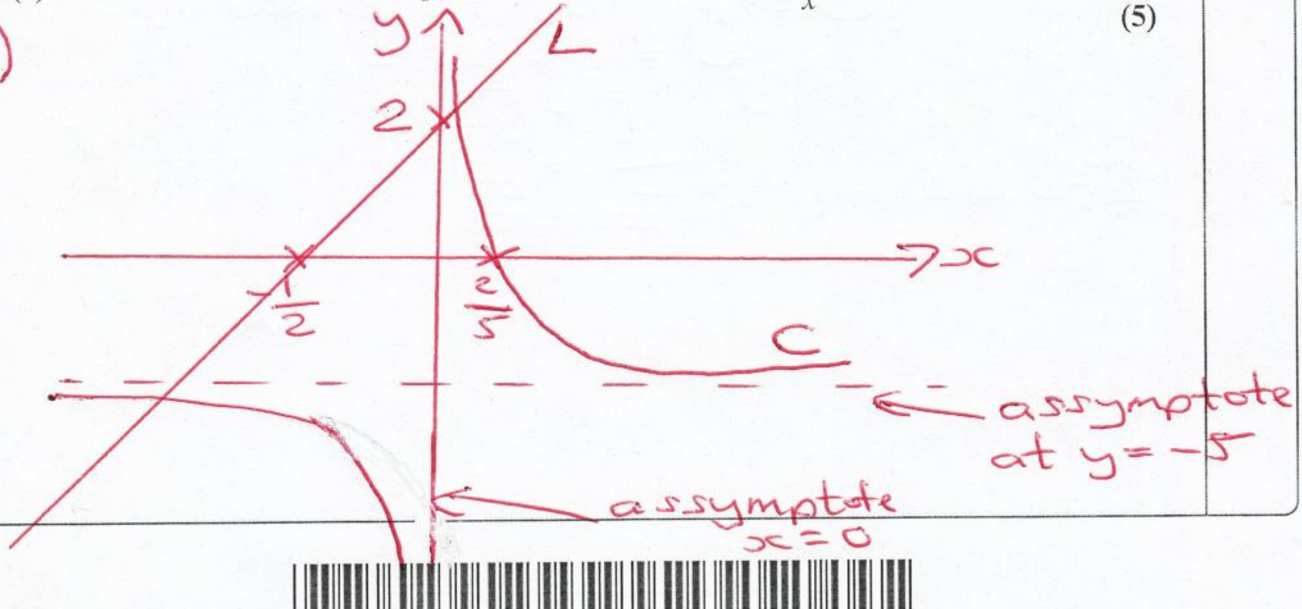
(b) Write down the equations of the asymptotes of the curve C.

(2)

(c) Find the coordinates of the points of intersection of $y = \frac{2}{x} - 5$ and $y = 4x + 2$

(5)

a)



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6b) $x = 0$ and $y = -5$

c) Solve simultaneous equations

$y = \frac{2}{x} - 5$ (1) and $y = 4x + 2$ (2)

set (1) = (2) to solve

$\frac{2}{x} - 5 = 4x + 2$

× through by x

$2 - 5x = 4x^2 + 2x$

$0 = 4x^2 + 2x + 5x - 2$

$0 = 4x^2 + 7x - 2$

$0 = (4x - 1)(x + 2)$

Either $4x - 1 = 0$

$x = \frac{1}{4}$

in (2) $y = 4 \times \frac{1}{4} + 2$

$y = 3$

$\underline{\underline{(\frac{1}{4}, 3)}}$

or $x + 2 = 0$

$x = -2$

in (2) $y = 4 \times -2 + 2$

$y = -6$

$\underline{\underline{(-2, -6)}}$

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10. $$4x^2 + 8x + 3 \equiv a(x + b)^2 + c$$

(a) Find the values of the constants a, b and c.

(3)

(b) On the axes on page 27, sketch the curve with equation $y = 4x^2 + 8x + 3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(4)

a) $4x^2 + 8x + 3$

$= 4\left(x^2 + 2x + \frac{3}{4}\right)$

$= 4\left[(x+1)^2 - 1 + \frac{3}{4}\right]$

$= 4\left[(x+1)^2 - \frac{1}{4}\right]$

$= 4(x+1)^2 - 1$

in form $a(x+b)^2 + c$

where $a = 4$, $b = 1$, $c = -1$

b) x^2 curve

minimum when $x + 1 = 0$

$x = -1$

and $y = -1$

y-axis, when $x = 0$, $y = 4(0+1)^2 - 1$

$= 4 - 1 = 3$

x-axis when $0 = 4(x+1)^2 - 1$

$1 = 4(x+1)^2$

$\frac{1}{4} = (x+1)^2$

square root both side

$x + 1 = \frac{1}{2}$ or $x + 1 = -\frac{1}{2}$

$x = -\frac{1}{2}$ $\qquad$ $x = -\frac{3}{2}$

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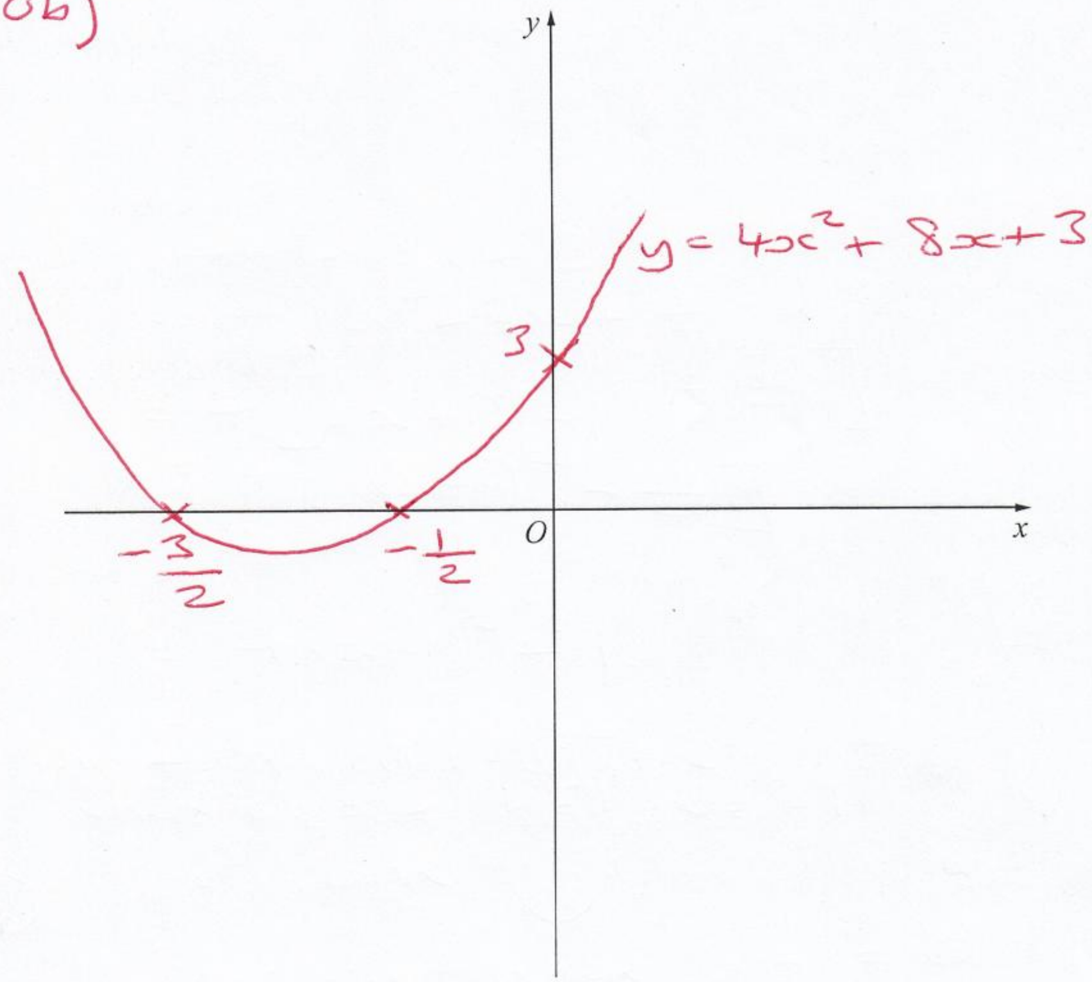
Question 10 continued

10b)



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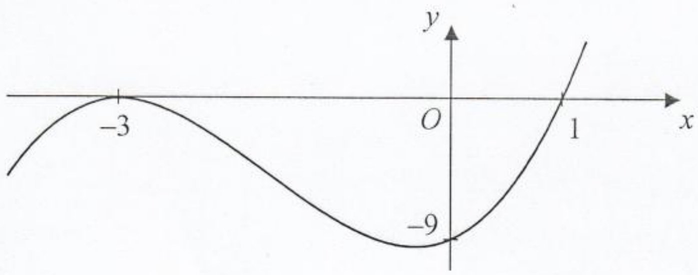


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x + 3)^2 (x - 1), \qquad x \in \mathbb{R}.$$

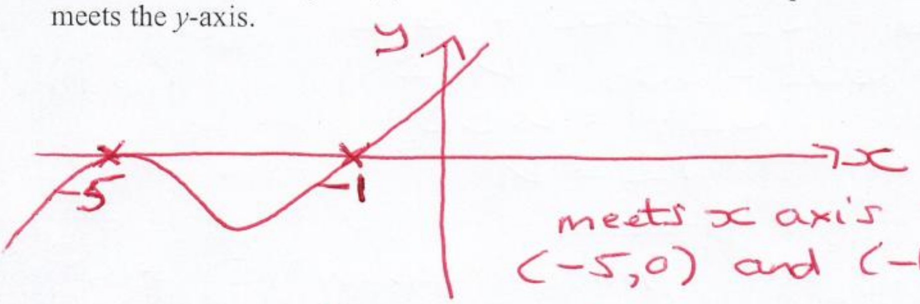
The curve crosses the x-axis at $(1, 0)$, touches it at $(-3, 0)$ and crosses the y-axis at $(0, -9)$

(a) In the space below, sketch the curve C with equation $y = \mathrm{f}(x + 2)$ and state the coordinates of the points where the curve C meets the x-axis. ↑ shifted 2 units left **(3)**

(b) Write down an equation of the curve C. **(1)**

(c) Use your answer to part (b) to find the coordinates of the point where the curve C meets the y-axis. **(2)**

a)



meets x axis at $(-5, 0)$ and $(-1, 0)$

b) $((x+2)+3)^2((x+2)-1)$ ← replace x with $(x+2)$ throughout

$= \underline{(x+5)^2(x+1)}$

c) meets y-axis at $x = 0$

$y = (0+5)^2(0+1) = 25$

Coordinates are $\underline{(0, 25)}$

P 4 1 8 0 2 A 0 1 4 2 8