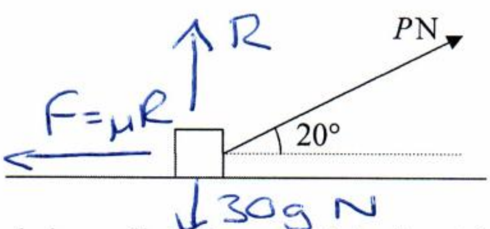
6.

Figure 3



$\mu = 0.4$

A box of mass 30 kg is being pulled along rough horizontal ground at a constant speed using a rope. The rope makes an angle of 20° with the ground, as shown in Figure 3. The coefficient of friction between the box and the ground is 0.4. The box is modelled as a particle and the rope as a light, inextensible string. The tension in the rope is P newtons.

(a) Find the value of P.

(8)

The tension in the rope is now increased to 150 N.

(b) Find the acceleration of the box.

(6)

a)

$R(\rightarrow)$ $\quad -F + P\cos 20^\circ = 0$

$-0.4R + P\cos 20^\circ = 0$

$P\cos 20^\circ = 0.4R$

$0.4R = P\cos 20^\circ$

$R = \dfrac{P\cos 20^\circ}{0.4}$

$R(\uparrow)$ $\quad P\sin 20^\circ + R - 30g = 0$

$P\sin 20^\circ + \dfrac{P\cos 20^\circ}{0.4} - 30g = 0$

$P\left(\sin 20^\circ + \dfrac{\cos 20^\circ}{0.4}\right) = 30 \times 9.8$

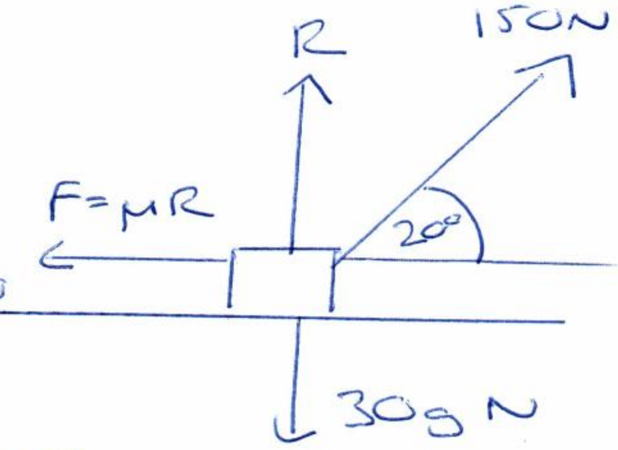
$P(2.6912517) = 294$

$P = \dfrac{294}{2.6912517} = 109.24285$

$P = 109\text{ N}$ (3 s.f.)

6 b (continued)

$R\ (\uparrow)^{+ve}$



$R + 150\sin 20° - 30g = 0$

$R = (30 \times 9.8) - 150\sin 20°$

Resolving along plane $(\longleftrightarrow)$ +ve

Resultant Force $= m \times a$

$150\cos 20° - F = 30 \times a$

$\therefore\ 150\cos 20° - \mu \times (30 \times 9.8 - 150\sin 20°) = 30a$

$\therefore\ \dfrac{150\cos 20° - 0.4(242.69698)}{30} = a$

$\therefore\ a = \dfrac{43.875102}{30} = 1.4625034$

$= 1.46\ ms^{-2}$ (3 sf)

4.

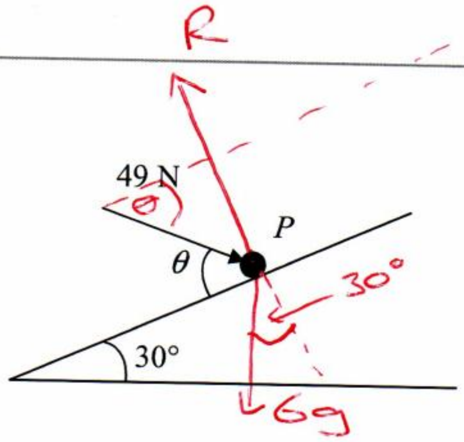


Figure 1

A particle P of mass 6 kg lies on the surface of a smooth plane. The plane is inclined at an angle of 30° to the horizontal. The particle is held in equilibrium by a force of magnitude 49 N, acting at an angle θ to the plane, as shown in Figure 1. The force acts in a vertical plane through a line of greatest slope of the plane.

(a) Show that $\cos\theta = \frac{3}{5}$.

(3)

(b) Find the normal reaction between P and the plane.

(4)

The direction of the force of magnitude 49 N is now changed. It is now applied horizontally to P so that P moves up the plane. The force again acts in a vertical plane through a line of greatest slope of the plane.

(c) Find the initial acceleration of P.

(4)

a) R (→) parallel to plane

$$49\cos\theta - 6g\sin 30° = 0$$

$$\cos\theta = \frac{6 \times 9.8 \times \sin 30°}{49}$$

$$\cos\theta = 0.6$$

$$\therefore \cos\theta = \frac{3}{5} \text{ (as required)}$$

b) R (↑) perpendicular to the plane

$$R - 49\sin\theta - 6g\cos 30° = 0$$

$$R = \left(49 \times \frac{4}{5}\right) + (6 \times 9.8 \times \cos 30°)$$

$$R = 39.2 + 50.922294$$

$$R = 90.122294$$

$$R = 90.1 \text{ N} \quad (3sf)$$

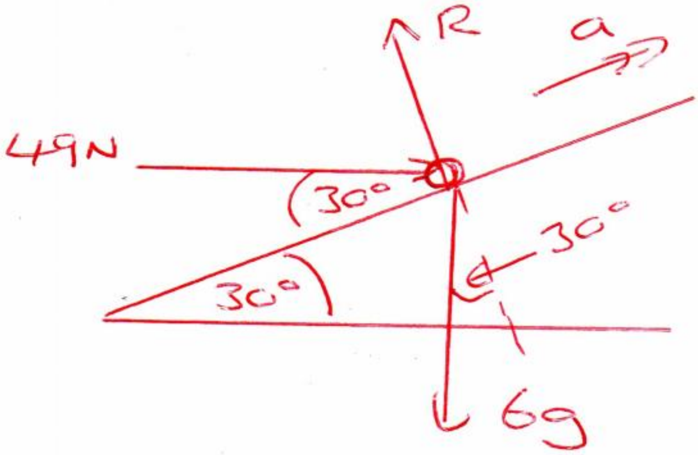
$\cos\theta = \frac{3}{5}$

$\sin\theta = \frac{4}{5}$

$\tan\theta =$

Leave blank

4c)



R ($\rightarrow$) parallel to plane

$$6 \times a = 49\cos 30° - 6g\sin 30°$$

$$a = \frac{49\cos 30° - 6 \times 9.8 \times \sin 30°}{6}$$

$$a = 2.1725408$$

$$a = 2.17\ ms^{-2} \quad (3\ sf)$$

Leave blank

5. A particle of mass 0.8 kg is held at rest on a rough plane. The plane is inclined at 30° to the horizontal. The particle is released from rest and slides down a line of greatest slope of the plane. The particle moves 2.7 m during the first 3 seconds of its motion. Find

(a) the acceleration of the particle,

(3)

(b) the coefficient of friction between the particle and the plane.

(5)

The particle is now held on the same rough plane by a horizontal force of magnitude X newtons, acting in a plane containing a line of greatest slope of the plane, as shown in Figure 3. The particle is in equilibrium and on the point of moving up the plane.

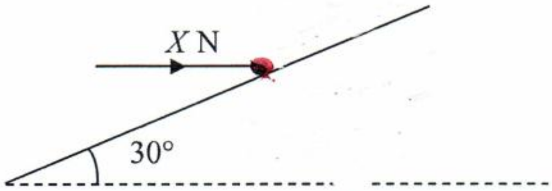


Figure 3

(c) Find the value of X.

(7)

a) $s = 2.7\text{m},\ u = 0\text{ms}^{-1},\ v = ?,\ a = ?\ t = 3$ seconds

$s = ut + \frac{1}{2}at^2$

$2.7 = 0 \times 3 + \frac{1}{2} \times a \times 3^2$

$a = \dfrac{2.7 \times 2}{3^2}$

$a = 0.6\text{ ms}^{-2}$

no motion perpendicular to plane

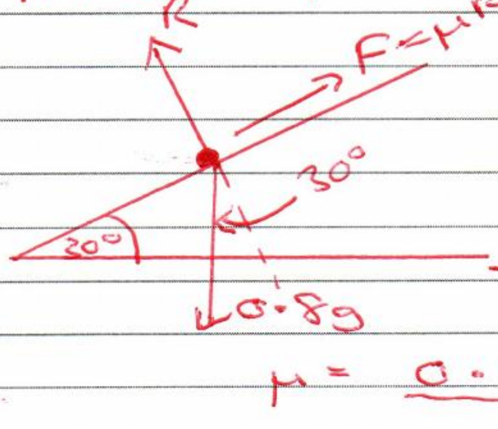
b) R (↖) perpendicular to plane

$R - 0.8g\cos 30° = 0$

$R = 0.8 \times 9.8 \times \cos 30°$

$R = 6.7896392\text{ N}$

Motion of particle is down the plane



R (↙) parallel to plane

$-\mu R + 0.8g\sin 30° = 0.8 \times 0.6$

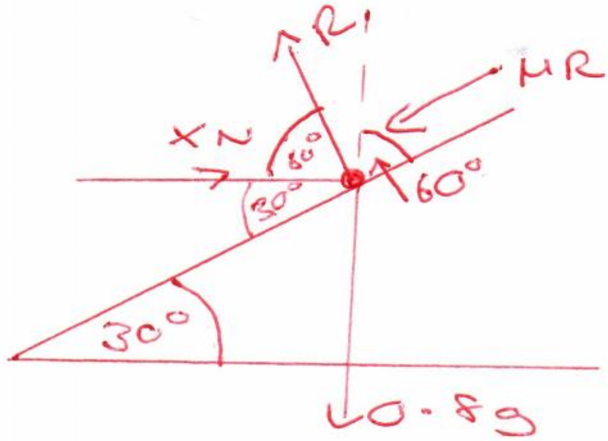
$\mu = \dfrac{0.8 \times 9.8 \times \sin 30° - (0.8 \times 0.6)}{6.7896392}$

$\mu = 0.5066543$

$\mu = 0.507$ (3sf)

mass × acceleration

5c)



in equilibrium on point of moving up the plane

R (↑)

$R \sin 60° - \mu R \cos 60° - 0.8g = 0$

$R(\sin 60° - 0.5066543 \cos 60°) = 0.8 \times 9.8$

$$R = \frac{0.8 \times 9.8}{\sin 60° - 0.5066543 \cos 60°}$$

$R = 12.795858$

R (→)

$X - R \cos 60° - \mu R \sin 60° = 0$

$X = 12.795858 \cos 60° + (0.5066543 \times 12.795858 \times \sin 60°)$

$X = 12.012438$

$X = 12.0$ N (3 sf)

Leave blank

6.

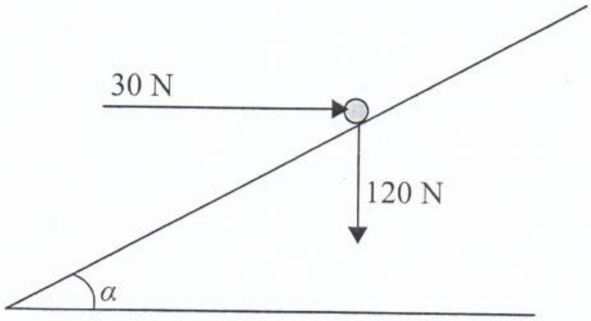


Figure 2

A particle of weight 120 N is placed on a fixed rough plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$.

The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

The particle is held at rest in equilibrium by a horizontal force of magnitude 30 N, which acts in the vertical plane containing the line of greatest slope of the plane through the particle, as shown in Figure 2.

(a) Show that the normal reaction between the particle and the plane has magnitude 114 N.
(4)

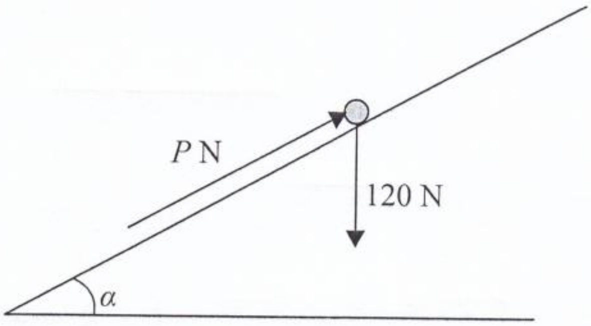


Figure 3

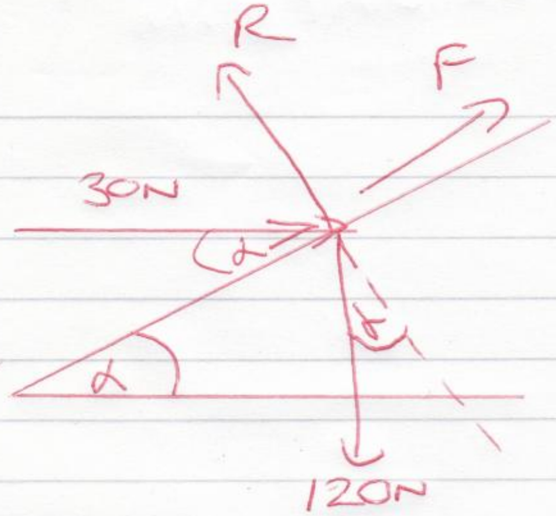
The horizontal force is removed and replaced by a force of magnitude P newtons acting up the slope along the line of greatest slope of the plane through the particle, as shown in Figure 3. The particle remains in equilibrium.

(b) Find the greatest possible value of P.
(8)

(c) Find the magnitude and direction of the frictional force acting on the particle when $P = 30$.
(3)

M1 JAN 2011

6) a)



$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$\mu = \frac{1}{2}$

R ($\rightarrow$) parallel to plane

$F + 30\cos\alpha - 120\sin\alpha = 0$

$F = 120\sin\alpha - 30\cos\alpha$

$F = (120 \times \frac{3}{5}) - (30 \times \frac{4}{5})$

$F = 48N$

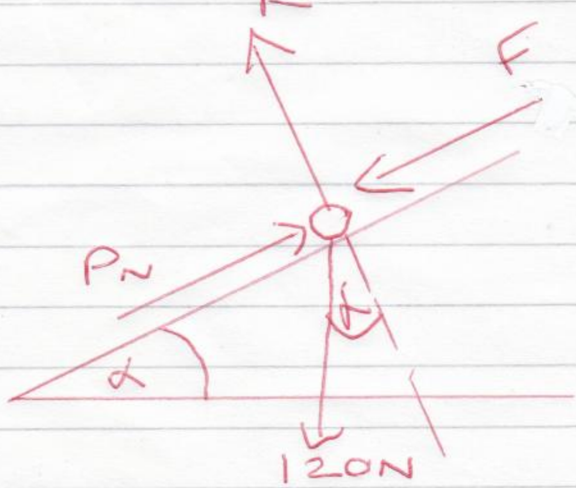
R ($\nwarrow$) perpendicular to plane

$R - 30\sin\alpha - 120\cos\alpha = 0$

$R = (30 \times \frac{3}{5}) + (120 \times \frac{4}{5})$

$R = 114N$

b)



M1 JAN 2011

6b) R ($\nwarrow$) perpendicular to plane

$$R - 120\cos\alpha = 0$$
$$R = 120 \times \frac{4}{5}$$
$$R = 96\text{N}$$

R ($\nearrow$) parallel to plane

$$P - F - 120\sin\alpha = 0$$
$$P - F = 120 \times \frac{3}{5}$$
$$P - F = 72 \quad (1)$$

Friction $F_{max} = \mu R$

$$F = \frac{1}{2} \times 96 = 48\text{N}$$

in (1) $P = 72 + 48$
$$= 120\text{N}$$

c) Using equation (1) when $P = 30\text{N}$

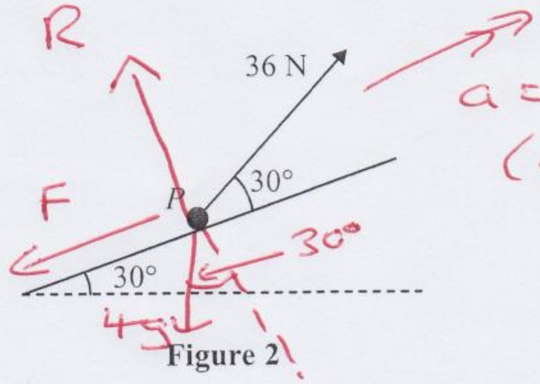
$$30 - F = 72$$
$$F = 30 - 72$$
$$= -42\text{N}$$

$F = 42\text{N}$ acting up the plane when $P = 30\text{N}$

M1 Jan 2012

Leave blank

8.



a = 0 m s^{-1} (constant speed)

Figure 2

A particle P of mass 4 kg is moving up a fixed rough plane at a constant speed of 16 m s^{-1} under the action of a force of magnitude 36 N. The plane is inclined at 30° to the horizontal. The force acts in the vertical plane containing the line of greatest slope of the plane through P, and acts at 30° to the inclined plane, as shown in Figure 2. The coefficient of friction between P and the plane is μ. Find

(a) the magnitude of the normal reaction between P and the plane,
(4)

(b) the value of μ.
(5)

The force of magnitude 36 N is removed.

(c) Find the distance that P travels between the instant when the force is removed and the instant when it comes to rest.
(5)

a) R ($\nearrow$) parallel to plane

$36\cos 30° - F - 4g\sin 30° = 0$ ①

R ($\nwarrow$) perpendicular to plane

$R + 36\sin 30° - 4g\cos 30° = 0$ ②

$F = \mu R$ ③

① gives $F = 36\cos 30° - 4 \times 9.8 \times \sin 30°$

$F = 11.576915$ N

② $R = 4 \times 9.8 \times \cos 30° - 36\sin 30°$

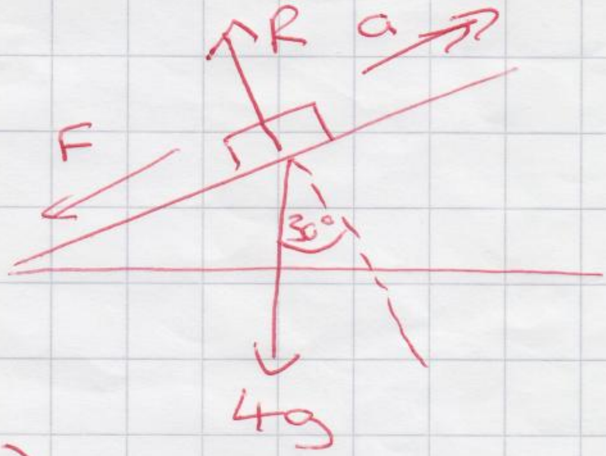
$= 15.948196$

b) ③ $\mu = \frac{F}{R} = \frac{11.576915}{15.948196} = 0.7259074$

$= 0.726$ (3 sf)

M1 Jan 2012

8c)



$R(\nearrow)$ parallel to plane

$-F - 4g\sin 30° = 4 \times a$ ①

$R(\quad)$ perpendicular to plane

$R - 4g\cos 30° = 0$

$R = 4 \times 9.8 \times \cos 30°$

$R = 33.948196\ N$ ②

$F = \mu R$ ③

$F = 0.7259074 \times 33.948196$

$F = 24.643247\ N$

in ① $a = \dfrac{-24.643247 - 4 \times 9.8 \times \sin 30°}{4}$

$a = -11.060812\ ms^{-2}$

$s = ?$

$u = 16\ ms^{-1}$

$v = 0\ ms^{-1}$

$a = -11.060812\ ms^{-2}$

$t = ?$

$v^2 = u^2 + 2as$

$s = \dfrac{v^2 - u^2}{2a}$

$s = \dfrac{0^2 - 16^2}{2 \times -11.060812}$

$s = 11.572387$

$= 11.6\ m\ (3\ sf)$

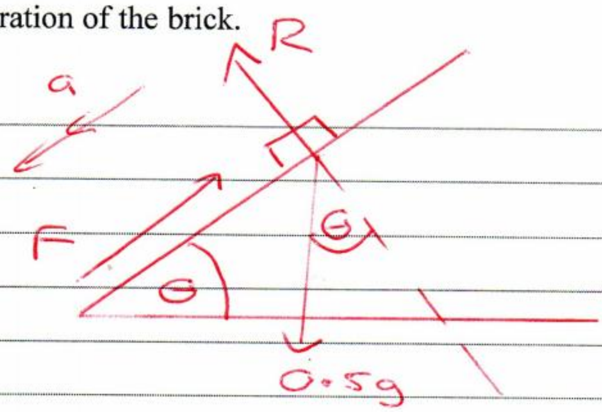
Distance travelled before coming to rest $= 11.6\ m\ (3sf)$

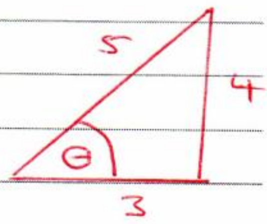
Leave blank

4. A small brick of mass 0.5 kg is placed on a rough plane which is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{4}{3}$, and released from rest. The coefficient of friction between the brick and the plane is $\frac{1}{3}$.

Find the acceleration of the brick.

(9)





$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

R (↖) perpendicular to plane – no acceleration in this direction

$R - 0.5g\cos\theta = 0$

$R = 0.5 \times 9.8 \times \frac{3}{5} = 2.94\,N$

$F = \mu R$

$\therefore F = \frac{1}{3} \times 2.94 = 0.98\,N$

R (↙) parallel to plane

$0.5 \times a = 0.5g\sin\theta - F$

$0.5 \times a = (0.5 \times 9.8 \times \frac{4}{5}) - 0.98$

$0.5a = 2.94$

$a = \frac{2.94}{0.5}$

$a = 5.88\,ms^{-2}$

May 2010

Leave blank

3.

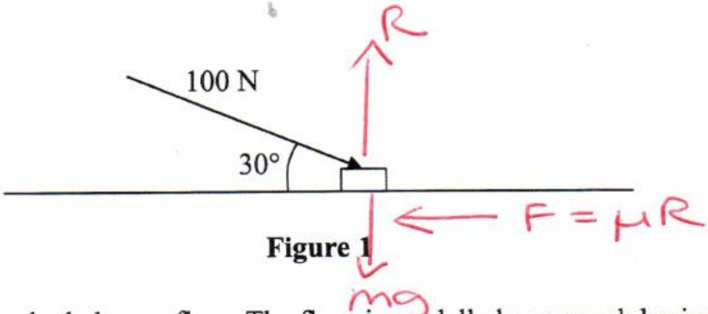


Figure 1

A small box is pushed along a floor. The floor is modelled as a rough horizontal plane and the box is modelled as a particle. The coefficient of friction between the box and the floor is $\frac{1}{2}$. The box is pushed by a force of magnitude 100 N which acts at an angle of 30° with the floor, as shown in Figure 1.

Given that the box moves with constant speed, find the mass of the box.

(7)

constant speed means acceleration is zero

$\mu = \frac{1}{2}$ coefficient of friction

let mass of particle be m

R (↑)

$R - 100\sin 30° - mg = 0$ (1)

R (→)

$100\cos 30° - F = 0$

$F = 100\cos 30°$ (2)

Substitute $F = \mu R$ in (1)

$\therefore R = \frac{F}{\mu} = \frac{100\cos 30°}{0.5}$

(1) gives $R - 100\sin 30° = mg$

$\frac{100\cos 30°}{0.5} - 100\sin 30° = 9.8 \times m$

$m = \frac{\left(\frac{100\cos 30°}{0.5}\right) - 100\sin 30°}{9.8}$

$m = 12.571949$

$m = 12.6$ kg (3sf)

N 3 5 3 9 0 A 0 6 2 8

May 2011

3.

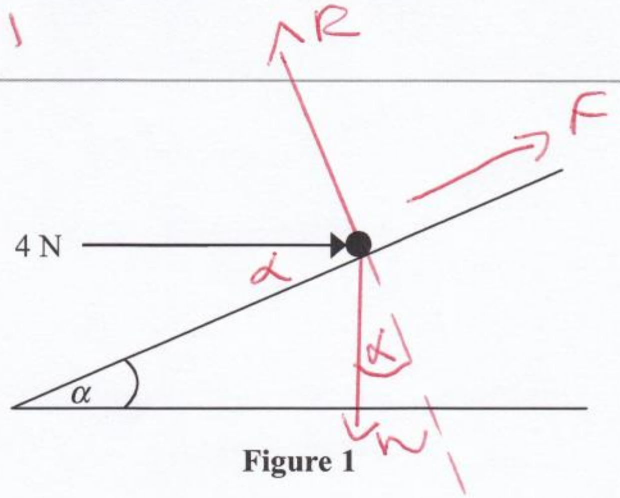


Figure 1

A particle of weight W newtons is held in equilibrium on a rough inclined plane by a horizontal force of magnitude 4 N. The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$, as shown in Figure 1.

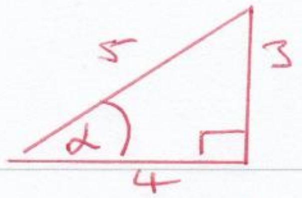
The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

Given that the particle is on the point of sliding down the plane,

(i) show that the magnitude of the normal reaction between the particle and the plane is 20 N,

(ii) find the value of W.

(9)



$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$ $\quad \cos\alpha = \frac{4}{5}$

(i) R (↖) perpendicular to plane

$R - W\cos\alpha - 4\sin\alpha = 0$ ①

R (↗) parallel to plane

$F - W\sin\alpha + 4\cos\alpha = 0$ ②

$F = \mu R$ ③

$F = \frac{1}{2}R$

sub in ②

$\frac{1}{2}R - \frac{3}{5}W + 4\times\frac{4}{5} = 0$

$R - \frac{6}{5}W + \frac{32}{5} = 0$

$R = \frac{6}{5}W - \frac{32}{5}$ ④

May 2011

Leave blank

Question 3 continued

Put in ①

$$\left(\frac{6}{5}W - \frac{32}{5}\right) - \left(W \times \frac{4}{5}\right) - \left(4 \times \frac{3}{5}\right) = 0$$

$$\frac{2}{5}W = \frac{32}{5} + \frac{12}{5}$$

$$2W = 32 + 12$$

(ii) $$2W = 44$$

$$W = 22\ \text{N}$$

(i) Put in ④

$$R = \left(\frac{6}{5} \times 22\right) - \frac{32}{5}$$

$R = 20$ N as required

MI MAY 2012

Leave blank

3.

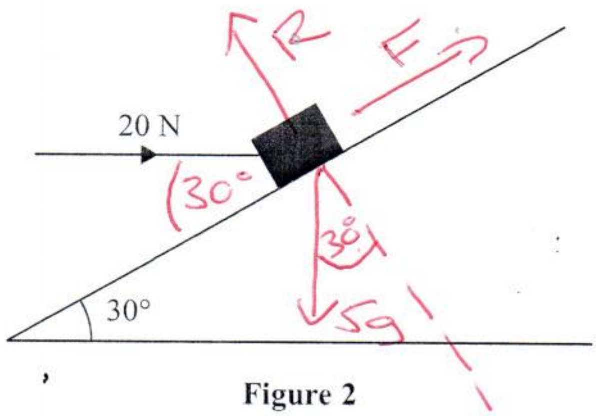


Figure 2

A box of mass 5 kg lies on a rough plane inclined at 30° to the horizontal. The box is held in equilibrium by a horizontal force of magnitude 20 N, as shown in Figure 2. The force acts in a vertical plane containing a line of greatest slope of the inclined plane.
The box is in equilibrium and on the point of moving down the plane. The box is modelled as a particle.

Find

(a) the magnitude of the normal reaction of the plane on the box, **(4)**

(b) the coefficient of friction between the box and the plane. **(5)**

a) R (→) parallel to plane

$F - 5g\sin 30° + 20\cos 30° = 0$ (1)

R (↖) perpendicular to plane

$R - 5g\cos 30° - 20\sin 30° = 0$ (2)

$R = 20\sin 30° + 5 \times 9.8 \times \cos 30°$

$R = 52.435245$ N

$R = 52.4$ N (3 sf)

b) $F = \mu R$ (3)

(2) gives $F = 5 \times 9.8 \times \sin 30° - 20\cos 30°$

$F = 7.1794919$

(3) gives $\mu = \frac{F}{R} = \frac{7.1794919}{52.435245}$

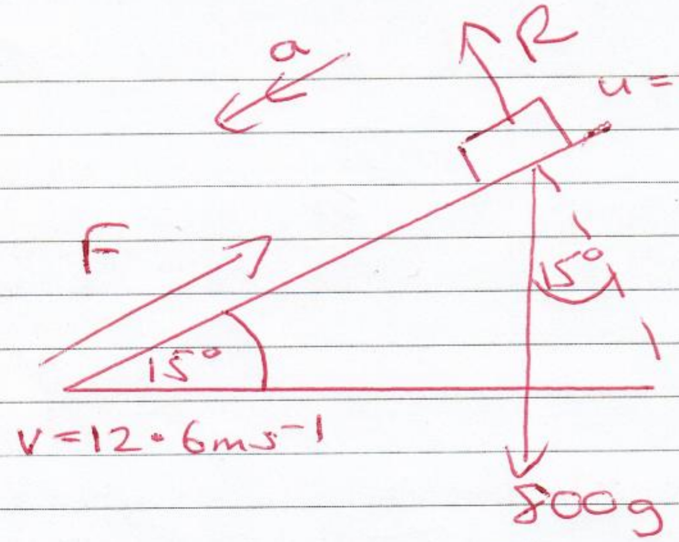
$\mu = 0.1369211$

$\mu = 0.137$ (3 sf)

Leave blank

4. A lifeboat slides down a straight ramp inclined at an angle of 15° to the horizontal. The lifeboat has mass 800 kg and the length of the ramp is 50 m. The lifeboat is released from rest at the top of the ramp and is moving with a speed of 12.6 m s^{-1} when it reaches the end of the ramp. By modelling the lifeboat as a particle and the ramp as a rough inclined plane, find the coefficient of friction between the lifeboat and the ramp.

(9)



First find acceleration of boat

$s = 50m$

$u = 0ms^{-1}$

$v = 12 \cdot 6ms^{-1}$

$a = ?$

$t = ?$

$$v^2 = u^2 + 2as$$

$$a = \frac{v^2 - u^2}{2s}$$

$$a = \frac{12 \cdot 6^2 - 0^2}{2 \times 50}$$

$$a = 1 \cdot 5876 ms^{-2}$$

Equation of motion parallel to plane

$R(\swarrow)$

$$800 \times 1 \cdot 5876 = 800g \sin 15° - F$$

$$F = 800 \times 9 \cdot 8 \times \sin 15° - 800 \times 1 \cdot 5876$$

$$F = 759 \cdot 06131 \text{ N}$$

Equation of motion perpendicular to plane

$R(\uparrow)$

$$800 \times 0 = R - 800g \times \cos 15°$$

$$R = 800 \times 9 \cdot 8 \times \cos 15°$$

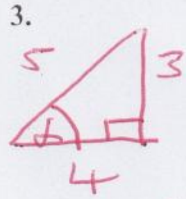
$$R = 7572 \cdot 8585 \text{ N}$$

$F = \mu R$ so $\mu = \frac{F}{R} = \frac{759 \cdot 06131}{7572 \cdot 8585}$

$\mu = 0 \cdot 1002344 = \underline{\underline{0 \cdot 100}}$ (3sf)

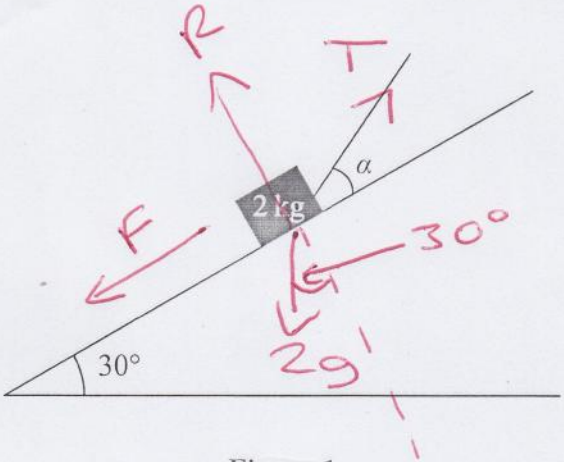
Leave blank

3.



$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$



$\mu = \frac{1}{3}$

Figure 1

A box of mass 2 kg is held in equilibrium on a fixed rough inclined plane by a rope. The rope lies in a vertical plane containing a line of greatest slope of the inclined plane. The rope is inclined to the plane at an angle α, where $\tan\alpha = \frac{3}{4}$, and the plane is at an angle of 30° to the horizontal, as shown in Figure 1. The coefficient of friction between the box and the inclined plane is $\frac{1}{3}$ and the box is on the point of slipping up the plane. By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.
(8)

Resolve ($\nearrow$) parallel to plane

$-F + T\cos\alpha - 2g\sin 30° = 0$ (1)

Resolve ($\nwarrow$) perpendicular to plane

$R - 2g\cos 30° + T\sin\alpha = 0$ (2)

$F = \mu R$

$F = \frac{1}{3}R$ (3)

Put (3) in (1) gives

$-\frac{1}{3}R + \frac{4}{5}T - g = 0$ (4)

(2) gives $R = 2g\cos 30° - \frac{3}{5}T$

sub R in (4) gives

$-\frac{1}{3}\left(2g\cos 30° - \frac{3}{5}T\right) + \frac{4}{5}T - g = 0$

$-5.56580326 + \frac{1}{5}T + \frac{4}{5}T = 9.8$

$T = 9.8 + 5.56580326$

$T = 15.458033$

$T = 15.5\text{ N}$ (3 sf)