

2. (a) Find the first 4 terms, in ascending powers of  $x$ , of the binomial expansion of  $(1-2x)^5$ . Give each term in its simplest form. (4)

(b) If  $x$  is small, so that  $x^2$  and higher powers can be ignored, show that

$$(1+x)(1-2x)^5 \approx 1 - 9x .$$

$$\begin{aligned} a) (1-2x)^5 &= 1 + {}^5C_1(-2x)^1 + {}^5C_2(-2x)^2 \\ &\quad + {}^5C_3(-2x)^3 + \dots \\ &= 1 + 5(-2x) + 10(-2)^2x^2 + 10(-2)^3x^3 + \dots \\ &= 1 - 10x + 40x^2 - 80x^3 + \dots \end{aligned} \quad (2)$$

$$\begin{aligned} b) (1+x)(1-2x)^5 &= (1+x)(1-10x + \dots) \\ &= 1 - 10x + x - 10x^2 + \dots \\ &= 1 - 9x - 10x^2 \\ &\approx 1 - 9x \end{aligned}$$

as  $x^2$  and higher powers can be ignored as  $x$  is small



3. (a) Find the first 4 terms of the expansion of  $\left(1 + \frac{x}{2}\right)^{10}$  in ascending powers of  $x$ , giving each term in its simplest form. (4)

- (b) Use your expansion to estimate the value of  $(1.005)^{10}$ , giving your answer to 5 decimal places. (3)

$$a) (1+a)^n = 1 + na + \frac{n(n-1)}{2!}a^2 + \frac{n(n-1)(n-2)}{3!}a^3 + \dots$$

$$\left(1 + \frac{x}{2}\right)^{10} = 1 + 10\left(\frac{x}{2}\right) + \frac{10(9)}{(2)(1)}\left(\frac{x}{2}\right)^2$$

$$+ \frac{10(9)(8)}{(3)(2)(1)}\left(\frac{x}{2}\right)^3 + \dots$$

$$= 1 + 5x + \frac{45}{4}x^2 + 15x^3 + \dots$$

$$b) (1.005)^{10}$$

$$\text{let } 1 + \frac{x}{2} = 1.005$$

$$\therefore \frac{x}{2} = 0.005$$

$$x = 0.01$$

$$\therefore (1.005)^{10} \approx 1 + 5(0.01) + \frac{45}{4}(0.01)^2$$

$$+ 15(0.01)^3$$

$$\approx 1 + 0.05 + 0.001125 + 0.000015 + \dots$$

$$\approx 1.05114 \quad (5 \text{dp})$$



1. Find the first 3 terms, in ascending powers of  $x$ , of the binomial expansion of  $(3 - 2x)^5$ , giving each term in its simplest form.

(4)

Binomial expansion (2 versions)

$$\textcircled{1} \quad (1+a)^n = 1 + na + \frac{n(n-1)}{2!} a^2 + \frac{n(n-1)(n-2)}{3!} a^3 \dots$$

$$\textcircled{2} \quad (a+b)^n = {}^n C_0 a^n b^0 + {}^n C_1 a^{n-1} b^1 + {}^n C_2 a^{n-2} b^2 \dots$$

Using \textcircled{2}

$$\begin{aligned} (3 - 2x)^5 &= {}^5 C_0 (3)^5 (-2x)^0 + {}^5 C_1 (3)^4 (-2x)^1 + {}^5 C_2 (3)^3 (-2x)^2 \\ &= 243 + 5(81)(-2x) + 10(27)(4)x^2 + \dots \\ &= 243 - 810x + 1080x^2 + \dots \end{aligned}$$

Q1

(Total 4 marks)



1. Find the first 3 terms, in ascending powers of  $x$ , of the binomial expansion of

$$(3 - x)^6$$

and simplify each term.

(4)

$$(a+b)^n = {}^n C_0 a^n b^0 + {}^n C_1 a^{n-1} b^1 + {}^n C_2 a^{n-2} b^2 \dots$$

$$(3-x)^6 = {}^6 C_0 (3)^6 (-x)^0 + {}^6 C_1 (3)^5 (-x)^1 + {}^6 C_2 (3)^4 (-x)^2$$

$$= 729 - 1458x + 1215x^2 + \dots$$

Q1

(Total 4 marks)



5. Given that  $\binom{40}{4} = \frac{40!}{4!b!}$ ,

(a) write down the value of  $b$ .

(1)

In the binomial expansion of  $(1+x)^{40}$ , the coefficients of  $x^4$  and  $x^5$  are  $p$  and  $q$  respectively.

- (b) Find the value of  $\frac{q}{p}$ .

(3)

a)  $b = 36$

b)  $(1+x)^{40}$

$$= 1 + 40x + \frac{40 \times 39}{1 \times 2} x^2 + \frac{40 \times 39 \times 38}{1 \times 2 \times 3} x^3$$

$$+ \frac{40 \times 39 \times 38 \times 37}{1 \times 2 \times 3 \times 4} x^4$$

$$+ \frac{40 \times 39 \times 38 \times 37 \times 36}{1 \times 2 \times 3 \times 4 \times 5} x^5$$

$p$   
 $q$

$$\frac{q}{p} = \frac{36}{5}$$



3. (a) Find the first 4 terms of the binomial expansion, in ascending powers of  $x$ , of

$$\left(1 + \frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

- (b) Use your expansion to estimate the value of  $(1.025)^8$ , giving your answer to 4 decimal places.

(3)

$$\begin{aligned}
 a) \quad & \left(1 + \frac{x}{4}\right)^8 \\
 &= 1 + \frac{8}{1} \times \left(\frac{x}{4}\right) + \frac{8 \times 7}{1 \times 2} \times \left(\frac{x}{4}\right)^2 \\
 &\quad + \frac{8 \times 7 \times 6}{1 \times 2 \times 3} \left(\frac{x}{4}\right)^3 + \dots \\
 &= 1 + 2x + \frac{28}{16}x^2 + \frac{56}{64}x^3 + \dots \\
 &= 1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3 + \dots
 \end{aligned}$$
  

$$\begin{aligned}
 b) \quad & \left(1 + 0.025\right)^8 \quad \text{let } x = 0.1 \\
 & \qquad \qquad \qquad \text{as } \frac{0.1}{4} = 0.025 \\
 & \left(1 + \frac{x}{4}\right)^8 = 1 + 2(0.1) + \frac{7}{4}(0.1)^2 + \frac{7}{8}(0.1)^3 \\
 &= 1 + 0.2 + 0.0175 + 8.75 \times 10^{-4} \\
 &= 1.218375 \\
 &= 1.2184 \quad (4dp)
 \end{aligned}$$



1. Find the first 3 terms, in ascending powers of  $x$ , of the binomial expansion of  $(2+x)^6$ , giving each term in its simplest form.

(4)

$$\begin{aligned}(2+x)^6 &= {}^6C_0(2^6)(x)^0 + {}^6C_1(2^5)(x)^1 + {}^6C_2(2^4)(x)^2 + \dots \\&= 1(64)(1) + 6(32)x + 15(16)x^2 + \dots \\&= 64 + 192x + 240x^2 + \dots\end{aligned}$$

Q1

(Total 4 marks)



3. (a) Find the first four terms, in ascending powers of  $x$ , in the binomial expansion of  $(1+kx)^6$ , where  $k$  is a non-zero constant.

(3)

Given that, in this expansion, the coefficients of  $x$  and  $x^2$  are equal, find

- (b) the value of  $k$ ,

(2)

- (c) the coefficient of  $x^3$ .

(1)

$$a) (1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3$$

but we have  $(1+kx)^6$

$$(1+kx)^6 = 1 + 6kx + \frac{6(5)(kx)^2}{2} + \frac{6(5)(4)(kx)^3}{3 \times 2 \times 1} + \dots$$

$$= 1 + 6kx + 15(kx)^2 + 20(kx)^3 + \dots$$

$$b) \text{ If coefficients of } x \text{ and } x^2 \text{ are equal}$$

$$6k^1 = 15k^2$$

$$k = \frac{6}{15}$$

$$k = \frac{2}{5}$$

c) Coefficient of  $x^3$  is

$$20 \times \left( \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \right) = \frac{32}{25}$$



3. (a) Find the first 4 terms, in ascending powers of  $x$ , of the binomial expansion of  $(1+ax)^{10}$ , where  $a$  is a non-zero constant. Give each term in its simplest form. (4)

Given that, in this expansion, the coefficient of  $x^3$  is double the coefficient of  $x^2$ ,

- (b) find the value of  $a$ .

a) 2 versions of Binomial Expansion (2)

$$\textcircled{1} \quad (a+b)^n = {}^n C_0 a^n b^0 + {}^n C_1 a^{n-1} b^1 + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_n a^0 b^n$$

$$\textcircled{2} \quad (1+b)^n = 1 + nb + \frac{n(n-1)b^2}{2!} + \frac{n(n-1)(n-2)}{3!} b^3 + \dots$$

Using  $\textcircled{1}$   $\rightarrow$   ${}^{10} C_0$  on calculator

$$(1+ax)^{10} = {}^{10} C_0 (1)^{10} (ax)^0 + {}^{10} C_1 (1)^9 (ax)^1 + {}^{10} C_2 (1)^8 (ax)^2 \\ + {}^{10} C_3 (1)^7 (ax)^3 + \dots$$

$$= 1 + 10ax + 45a^2x^2 + 120a^3x^3 + \dots$$

or

Using  $\textcircled{2}$   $\rightarrow$  b

$$(1+ax)^{10} = 1 + 10(ax) + \frac{10(9)}{(2)(1)} (ax)^2 + \frac{10(9)(8)}{(3)(2)(1)} (ax)^3 + \dots$$

$$= 1 + 10ax + 45a^2x^2 + 120a^3x^3 + \dots$$

b)  $120a^3 = 2(45a^2)$

$$\therefore 120a^3 = 90a^2$$

$$\therefore 120a^3 - 90a^2 = 0$$

$$30a^2(4a-3) = 0$$

$$\therefore a = 0 \quad \text{or} \quad 4a-3 = 0$$

$$\therefore a = 0 \quad \text{or} \quad a = \frac{3}{4}$$

Since  $a > 0$

$$\therefore a = \frac{3}{4}$$



2. (a) Find the first 3 terms, in ascending powers of  $x$ , of the binomial expansion of

$$(2 + kx)^7$$

where  $k$  is a constant. Give each term in its simplest form.

(4)

Given that the coefficient of  $x^2$  is 6 times the coefficient of  $x$ ,

- (b) find the value of  $k$ .

a)  $(a + bx)^n = a^n + {}^n C_1 a^{n-1} b x + {}^n C_2 a^{n-2} b^2 x^2 + \dots$

$$\begin{aligned} (2 + kx)^7 &= 2^7 + {}^7 C_1 2^6 kx + {}^7 C_2 2^5 k^2 x^2 + \dots \\ &= 128 + 7(2^6) kx + 21(2^5) k^2 x^2 + \dots \\ &= 128 + 448 kx + 672 k^2 x^2 + \dots \end{aligned}$$

b) If coefficient of  $x^2$  is 6 times coefficient of  $x$

$$\begin{aligned} 672k^2 &= 6 \times 448k \\ k &= \frac{6 \times 448}{672} \end{aligned}$$

$$k = 4$$

4. (a) Find the first 4 terms, in ascending powers of  $x$ , of the binomial expansion of  $(1+ax)^7$ , where  $a$  is a constant. Give each term in its simplest form. (4)

Given that the coefficient of  $x^2$  in this expansion is 525,

- (b) find the possible values of  $a$ . (2)

$$\begin{aligned} a) (1+ax)^7 &= 1 + 7(ax) + \frac{7 \cdot 6}{1 \cdot 2} (ax)^2 \\ &\quad + \frac{7 \cdot 6 \cdot 5}{1 \cdot 2 \cdot 3} (ax)^3 + \dots \end{aligned}$$

$$= 1 + 7ax + 21a^2x^2 + 35a^3x^3 + \dots$$

$$\begin{aligned} b) 21a^2 &= 525 \\ a^2 &= \frac{525}{21} \\ a^2 &= 25 \end{aligned}$$

$$a = 5 \text{ or } a = -5$$



2. (a) Find the first 3 terms, in ascending powers of  $x$ , of the binomial expansion of

$$(3+bx)^5$$

where  $b$  is a non-zero constant. Give each term in its simplest form.

(4)

Given that, in this expansion, the coefficient of  $x^2$  is twice the coefficient of  $x$ ,

- (b) find the value of  $b$ .

$$1 + n\alpha + \frac{n(n-1)}{2!} \alpha^2 + \dots \quad (2)$$

$$\text{a) } (3+bx)^5$$

$$= 3^5 \left(1 + \frac{b}{3}\alpha\right)^5$$

$$= 3^5 \left[ 1 + 5\left(\frac{b}{3}\alpha\right) + \frac{5 \times 4}{1 \times 2} \left(\frac{b}{3}\alpha\right)^2 + \dots \right]$$

$$= 243 \left[ 1 + \frac{5b}{3}\alpha + \frac{10}{9} b^2 \alpha^2 + \dots \right]$$

$$= 243 + 405b\alpha + 270b^2\alpha^2 + \dots$$

- b) If coefficient of  $\alpha^2$  is twice coefficient of  $\alpha$

$$2 \times 405b = 270b^2$$

$$b = \frac{2 \times 405}{270}$$

$$b = 3$$



1. Find the first 3 terms, in ascending powers of  $x$ , of the binomial expansion of

$$(2 - 3x)^5$$

giving each term in its simplest form.

(4)

$$\begin{aligned}
 & 2^5 + {}^5C_1 2^4 (-3x)^1 + {}^5C_2 2^3 (-3x)^2 \\
 & \quad + \dots \\
 = & 32 + 5 \times 16 x (-3x) + 10 \times 8 \times 9 x^2 + \dots \\
 = & 32 - 240x + 720x^2 + \dots
 \end{aligned}$$

Q1

(Total 4 marks)



1. Find the first 3 terms, in ascending powers of  $x$ , in the binomial expansion of

$$(2 - 5x)^6$$

Give each term in its simplest form.

(4)

$${}^6C_0 2^6 + {}^6C_1 2^5 (-5x)^1 + {}^6C_2 2^4 (-5x)^2 + \dots$$

$$= 2^6 + 6 \times 32 (-5x) + 15 \times 2^4 (-5x)^2 + \dots$$

$$= 64 - 960x + 6000x^2 + \dots$$

Q1

(Total 4 marks)



2. (a) Use the binomial theorem to find all the terms of the expansion of

$$(2 + 3x)^4$$

Give each term in its simplest form.

(4)

- (b) Write down the expansion of

$$(2 - 3x)^4$$

in ascending powers of  $x$ , giving each term in its simplest form.

$$(a+bx)^n = {}^n C_0 a^n + {}^n C_1 a^{n-1} b x + {}^n C_2 a^{n-2} b^2 x^2 + \dots \quad (1)$$

a)  $(2 + 3x)^4$

$$= {}^4 C_0 2^4 + {}^4 C_1 2^3 \times 3x + {}^4 C_2 2^2 (3x)^2 \\ + {}^4 C_3 2 \times (3x)^3 + {}^4 C_4 (3x)^4$$

$$= 16 + (4 \times 8 \times 3x) + (6 \times 4 \times 9x^2) \\ + (4 \times 2 \times 27x^3) + 81x^4$$

$$= \underline{\underline{16 + 96x + 216x^2 + 216x^3 + 81x^4}}$$

b)  $16 - 96x + 216x^2 - 216x^3 + 81x^4$

*these "odd" powers of  $x$   
change sign*

