

January 2007
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1.	<p>** represents a constant</p> $f(x) = (2 - 5x)^{-2} = \underline{(2)^{-2}} \left(1 - \frac{5x}{2}\right)^{-2} = \frac{1}{4} \left(1 - \frac{5x}{2}\right)^{-2}$ <p>$= \frac{1}{4} \left\{ 1 + (-2)(**x) + \frac{(-2)(-3)}{2!} (**x)^2 + \frac{(-2)(-3)(-4)}{3!} (**x)^3 + \dots \right\}$</p> <p>$= \frac{1}{4} \left\{ 1 + (-2)\left(\frac{-5x}{2}\right) + \frac{(-2)(-3)}{2!} \left(\frac{-5x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{3!} \left(\frac{-5x}{2}\right)^3 + \dots \right\}$</p> <p>$= \frac{1}{4} \left\{ 1 + 5x; + \frac{75x^2}{4} + \frac{125x^3}{2} + \dots \right\}$</p> <p>$= \frac{1}{4} + \frac{5x}{4}; + \frac{75x^2}{16} + \frac{125x^3}{8} + \dots$</p> <p>$= \frac{1}{4} + 1\frac{1}{4}x; + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \dots$</p> <p>Takes 2 outside the bracket to give any of $(2)^{-2}$ or $\frac{1}{4}$. B1</p> <p>Expands $(1 + **x)^{-2}$ to give an unsimplified $1 + (-2)(**x)$; M1</p> <p>A correct unsimplified $\{\dots\}$ expansion with candidate's $(**x)$ A1</p> <p>Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$; Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$ A1</p>	[5]

5 marks

Question Number	Scheme	Marks
<p><i>Aliter</i> 1. Way 2</p> $f(x) = (2 - 5x)^{-2}$ $= \left\{ (2)^{-2} + (-2)(2)^{-3}(**x); + \frac{(-2)(-3)}{2!}(2)^{-4}(**x)^2 \right.$ $\quad \left. + \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(**x)^3 + \dots \right\}$ $= \left\{ (2)^{-2} + (-2)(2)^{-3}(-5x); + \frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2 \right.$ $\quad \left. + \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(-5x)^3 + \dots \right\}$ $= \left\{ \frac{1}{4} + (-2)\left(\frac{1}{8}\right)(-5x); + (3)\left(\frac{1}{16}\right)(25x^2) \right.$ $\quad \left. + (-4)\left(\frac{1}{16}\right)(-125x^3) + \dots \right\}$ $= \frac{1}{4} + \frac{5x}{4}; + \frac{75x^2}{16} + \frac{125x^3}{8} + \dots$ $= \frac{1}{4} + 1\frac{1}{4}x; + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \dots$	<p>$\frac{1}{4}$ or $(2)^{-2}$</p> <p>Expands $(2 - 5x)^{-2}$ to give an unsimplified $(2)^{-2} + (-2)(2)^{-3}(**x);$</p> <p>A correct unsimplified $\{\dots\}$ expansion with candidate's $(**x)$</p> <p>Anything that cancels to $\frac{1}{4} + \frac{5x}{4};$</p> <p>Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1;</p> <p>A1</p>

[5]

5 marks

Attempts using Maclaurin expansions need to be referred to your team leader.

2. (a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \quad |x| < \frac{8}{3},$$

in ascending powers of x , up to and including the term in x^3 , giving each term as a simplified fraction.

(5)

- (b) Use your expansion, with a suitable value of x , to obtain an approximation to $\sqrt[3]{7.7}$. Give your answer to 7 decimal places.

$$(8-3x)^{\frac{1}{3}} = 8^{\frac{1}{3}} \left(1 - \frac{3x}{8}\right)^{\frac{1}{3}} \quad (2)$$

$$= 2 \left(1 - \frac{3x}{8}\right)^{\frac{1}{3}}$$

$$= 2 \left(1 + \left(\frac{1}{3}\right)\left(-\frac{3x}{8}\right) + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!} \left(-\frac{3x}{8}\right)^2 - \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{3!} \left(-\frac{3x}{8}\right)^3\right)$$

$$= 2 \left(1 - \frac{1}{8}x - \frac{1}{64}x^2 - \frac{5}{1536}x^3\right)$$

$$= 2 - \frac{1}{4}x - \frac{1}{32}x^2 - \frac{5}{768}x^3$$

b) $\sqrt[3]{7.7} \quad (8-3x)^{\frac{1}{3}} \quad \text{when } x = 0.1$

$$\sqrt[3]{7.7} = 2 - \frac{1}{4}(0.1) - \frac{1}{32}(0.1)^2 - \frac{5}{768}(0.1)^3$$

$$= \cancel{1.9746800}$$

$$= 1.9746800$$



3.

$$f(x) = \frac{27x^2 + 32x + 16}{(3x+2)^2(1-x)}, \quad |x| < \frac{2}{3}$$

Given that $f(x)$ can be expressed in the form

$$f(x) = \frac{A}{(3x+2)} + \frac{B}{(3x+2)^2} + \frac{C}{(1-x)},$$

- (a) find the values of B and C and show that $A = 0$.

(4)

- (b) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x , up to and including the term in x^2 . Simplify each term.

(6)

- (c) Find the percentage error made in using the series expansion in part (b) to estimate the value of $f(0.2)$. Give your answer to 2 significant figures.

(4)

a) $27x^2 + 32x + 16 \equiv A(3x+2)(1-x) + B(1-x) + C(3x+2)^2$

When $x = 1$ $75 = 25C$

$C = 3$

$x = -\frac{2}{3}$ $\frac{20}{3} = \frac{5}{3}B$

$B = 4$

Choose any value eg. $x = 0$ $16 = 2A + B + 4C$

$(B = 4 \quad C = 3)$

$16 = 2A + 4 + 12$

$A = 0$



$$b) f(x) = \frac{4}{(3x+2)^2} + \frac{3}{1-x}$$

$$= 4(3x+2)^{-2} + 3(1-x)^{-1}$$

$$= 4(2+3x)^{-2} + 3(1-x)^{-1}$$

$$= 4 \cdot 2^{-2} \left(1 + \frac{3x}{2}\right)^{-2} + 3(1-x)^{-1}$$

$$= \left(1 + \frac{3x}{2}\right)^{-2} + 3(1-x)^{-1}$$

$$= 1 + (-2)\left(\frac{3x}{2}\right) + \frac{(-2)(-3)}{2!} \times \left(\frac{3x}{2}\right)^2 + 3 \int 1 + (-1)(-x) + \frac{(-1)(-2)(-x)}{2!}$$

$$= 1 - 3x + \frac{27x^2}{4} + 3 + 3x + 3x^2$$

$$= 4 + \frac{39}{4}x^2$$

Binomial
Expansion

(c) Actual

$$\text{Substitute } 0.2 \text{ into } f(x) \quad f(0.2) = \frac{2935}{676}$$

$$\text{Estimate: Substitute } 0.2 \text{ into } 4 + \frac{39}{4}x^2 = \frac{439}{100}$$

$$\% \text{ Error} = \frac{\text{difference}}{\text{original}} \times 100$$

\rightarrow ignore negative sign.
take modulus.

$$= \left(\frac{4.3417 - 4.39}{4.3417} \right) \times 100$$

$$= 1.112$$

$$= 1.1\% \text{ (2 s.f.)}$$

1. (a) Find the binomial expansion of

$$\sqrt{1-8x}, \quad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3 , simplifying each term.

(6)

- (b) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{1-8x}$ is $\frac{\sqrt{23}}{5}$.

(2)

- (c) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

(3)

$$\text{a) } (1-8x)^{1/2}$$

$$\begin{aligned} &= 1 + \left(\frac{1}{2}\right)(-8x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2} (-8x)^2 + \underbrace{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}_{3!} (-8x)^3 \\ &= 1 - 4x - 8x^2 - 32x^3 \end{aligned}$$

$$\begin{aligned} \text{(b) } \sqrt{1-8x} &= \sqrt{1-\frac{8}{100}} \\ &= \sqrt{\frac{92}{100}} = \sqrt{\frac{23}{25}} = \frac{\sqrt{23}}{5} \end{aligned}$$

$$\begin{aligned} \text{(c) } 1 - 4x - 8x^2 - 32x^3 &= 1 - 4(0.01) - 8(0.01)^2 - 32(0.01)^3 \\ &= 0.959168 \end{aligned}$$

$$\begin{aligned} \sqrt{23} &= 5 \times 0.959168 \\ &= 4.79584 \end{aligned}$$

3. (a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5}$$

in ascending powers of x , up to and including the term in x^2 , giving each term as a simplified fraction.

(5)

Given that the binomial expansion of $\frac{2+kx}{(2-5x)^2}, \quad |x| < \frac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \dots$$

- (b) find the value of the constant k ,

(2)

- (c) find the value of the constant A .

(2)

$$\text{a) } (2-5x)^{-2} = 2^{-2} \left(1 - \frac{5}{2}x\right)^{-2}$$

$$= \frac{1}{4} \left[1 + (-2)\left(-\frac{5}{2}x\right) + \underbrace{(-2)(-3)\left(-\frac{5}{2}x\right)^2}_{2} + \dots \right]$$

$$= \frac{1}{4} \left[1 + 5x + \frac{75}{4}x^2 \right]$$

$$= \frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2$$

$$\text{(b) } (2+kx)(\frac{1}{4} + \frac{5}{4}x + \frac{75}{16}x^2) \equiv \frac{1}{2} + \frac{7}{4}x + Ax^2$$

$$\text{xc terms: } \frac{1}{4}kx + 2\left(\frac{5}{4}x\right) = \frac{7}{4}x$$

$$\Rightarrow k + 10 = 7 \\ k = -3$$



Question 3 continued

(c) x^2 terms :

$$\frac{5kx^2}{4} + \frac{150x^2}{16} = Ax^2$$

$$\text{but } k = -3$$

$$\frac{-15}{4} + \frac{150}{16} = A$$

$$A = \frac{45}{8}$$



P 4 0 0 8 5 A 0 7 2 8

Final Version

**June 2005
6666 Core C4
Mark Scheme**

Question Number	Scheme	Marks
1.	$ \begin{aligned} (4-9x)^{\frac{1}{2}} &= 2\left(1-\frac{9x}{4}\right)^{\frac{1}{2}} \\ &= 2\left(1+\frac{\frac{1}{2}}{1}\left(-\frac{9x}{4}\right) + \frac{\frac{1}{2}(-\frac{1}{2})}{1.2}\left(-\frac{9x}{4}\right)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1.2.3}\left(-\frac{9x}{4}\right)^3 + \dots\right) \\ &= 2\left(1-\frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \dots\right) \\ &= 2 - \frac{9}{4}x, -\frac{81}{64}x^2, -\frac{729}{512}x^3 + \dots \end{aligned} $	B1 M1 A1, A1, A1 [5]
	<p><i>Note</i> The M1 is gained for $\frac{\frac{1}{2}(-\frac{1}{2})}{1.2}(\dots)^2$ or $\frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{1.2.3}(\dots)^3$</p> <p><i>Special Case</i></p> <p>If the candidate reaches $= 2\left(1-\frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \dots\right)$ and goes no further allow A1 A0 A0</p>	

Question Number	Scheme	Marks
2. (a)	$3x - 1 = A(1 - 2x) + B$ <p>Let $x = \frac{1}{2}$; $\frac{3}{2} - 1 = B \Rightarrow B = \frac{1}{2}$</p> <p>Equate x terms; $3 = -2A \Rightarrow A = -\frac{3}{2}$</p> <p>(No working seen, but A and B correctly stated \Rightarrow award all three marks. If one of A or B correctly stated give two out of the three marks available for this part.)</p>	Considers this identity and either substitutes $x = \frac{1}{2}$, equates coefficients or solves simultaneous equations M1 $A = -\frac{3}{2}; B = \frac{1}{2}$ A1; A1 [3]
(b)	$f(x) = -\frac{3}{2}(1 - 2x)^{-1} + \frac{1}{2}(1 - 2x)^{-2}$ $= -\frac{3}{2} \left\{ 1 + (-1)(-2x) + \frac{(-1)(-2)}{2!} (-2x)^2 + \frac{(-1)(-2)(-3)}{3!} (-2x)^3 + \dots \right\}$ $+ \frac{1}{2} \left\{ 1 + (-2)(-2x) + \frac{(-2)(-3)}{2!} (-2x)^2 + \frac{(-2)(-3)(-4)}{3!} (-2x)^3 + \dots \right\}$ $= -\frac{3}{2} \{1 + 2x + 4x^2 + 8x^3 + \dots\} + \frac{1}{2} \{1 + 4x + 12x^2 + 32x^3 + \dots\}$ $= -1 - x + 0x^2 + 4x^3$	Moving powers to top on any one of the two expressions M1 Either $1 \pm 2x$ or $1 \pm 4x$ from either first or second expansions respectively dM1; Ignoring $-\frac{3}{2}$ and $\frac{1}{2}$, any one correct $\{ \dots \}$ expansion. A1 Both $\{ \dots \}$ correct. A1 $-1 - x; (0x^2) + 4x^3$ A1; A1 [6]

Beware: In part (a) take care to spot that $A = -\frac{3}{2}$ and $B = \frac{1}{2}$ are the right way around.

Beware: In ePEN, make sure you aware the marks correctly in part (a). The first A1 is for $A = -\frac{3}{2}$ and the second A1 is for $B = \frac{1}{2}$.

Beware: If a candidate uses a method of long division please escalate this to your team leader.

Question Number	Scheme	Marks
Aliter 2. (b) Way 2	$f(x) = (3x - 1)(1 - 2x)^{-2}$ $= (3x - 1) \times \left(1 + (-2)(-2x) + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \dots \right)$ $= (3x - 1)(1 + 4x + 12x^2 + 32x^3 + \dots)$ $= \underline{3x + 12x^2 + 36x^3} - 1 - 4x - 12x^2 - 32x^3 + \dots$ $= -1 - x + 0x^2 + 4x^3$	Moving power to top dM1; Ignoring $(3x - 1)$, correct (\dots) expansion <u>Correct expansion</u> -1 - x ; $(0x^2) + 4x^3$ A1; A1 [6]
Aliter 2. (b) Way 3	Maclaurin expansion $f(x) = -\frac{3}{2}(1 - 2x)^{-1} + \frac{1}{2}(1 - 2x)^{-2}$ $f'(x) = -3(1 - 2x)^{-2} + 2(1 - 2x)^{-3}$ $f''(x) = -12(1 - 2x)^{-3} + 12(1 - 2x)^{-4}$ $f'''(x) = -72(1 - 2x)^{-4} + 96(1 - 2x)^{-5}$ $\therefore f(0) = -1, f'(0) = -1, f''(0) = 0 \text{ and } f'''(0) = 24$ gives $f(x) = -1 - x + 0x^2 + 4x^3 + \dots$	Bringing both powers to top Differentiates to give $a(1 - 2x)^{-2} \pm b(1 - 2x)^{-3};$ $-3(1 - 2x)^{-2} + 2(1 - 2x)^{-3}$ Correct $f''(x)$ and $f'''(x)$ -1 - x ; $(0x^2) + 4x^3$ A1; A1 [6]

Question Number	Scheme	Marks
Aliter		
2. (b)	$f(x) = -3(2 - 4x)^{-1} + \frac{1}{2}(1 - 2x)^{-2}$	Moving powers to top on any one of the two expressions
Way 4	$= -3 \left\{ (2)^{-1} + (-1)(2)^{-2}(-4x) + \frac{(-1)(-2)}{2!}(2)^{-3}(-4x)^2 \right.$ $\quad \left. + \frac{(-1)(-2)(-3)}{3!}(2)^{-4}(-4x)^3 + \dots \right\}$ $+ \frac{1}{2} \left\{ 1 + (-2)(-2x) + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \dots \right\}$ $= -3 \left\{ \frac{1}{2} + x + 2x^2 + 4x^3 + \dots \right\} + \frac{1}{2} \left\{ 1 + 4x + 12x^2 + 32x^3 + \dots \right\}$ $= -1 - x ; + 0x^2 + 4x^3$	Either $\frac{1}{2} \pm x$ or $1 \pm 4x$ from either first or second expansions respectively dM1; Ignoring -3 and $\frac{1}{2}$, any one correct $\{\dots\}$ expansion. Both $\{\dots\}$ correct.
		A1
		A1
		A1; A1
		[6]

1.

$$f(x) = (3+2x)^{-3}, \quad |x| < \frac{3}{2}.$$

Find the binomial expansion of $f(x)$, in ascending powers of x , as far as the term in x^3 .

Give each coefficient as a simplified fraction.

(5)

$$f(x) = 3^{-3} \left(1 + \frac{2}{3}x\right)^{-3}$$

$$= \frac{1}{27} \left\{ 1 + (-3)\left(\frac{2}{3}x\right) + \frac{(-3)(-4)}{2!} \left(\frac{2}{3}x\right)^2 + \frac{(-3)(-4)(-5)}{3!} \left(\frac{2}{3}x\right)^3 \dots \right.$$

$$= \frac{1}{27} \left\{ 1 - 2x + \frac{8x^2}{3} - \frac{80x^3}{27} \dots \right.$$

$$= \frac{1}{27} - \frac{2}{27}x + \frac{8}{81}x^2 - \frac{80}{729}x^3 \dots$$



5. (a) Expand $\frac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2 . Simplify each term.

(5)

- (b) Hence, or otherwise, find the first 3 terms in the expansion of $\frac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of x .

(4)

$$5(a) \quad (4-3x)^{-1/2}$$

$$= 4^{-1/2} \left(1 - \frac{3x}{4}\right)^{-1/2}$$

$$= \frac{1}{2} \left(1 - \frac{3x}{4}\right)^{-1/2}$$

$$= \frac{1}{2} \left(1 + \left(-\frac{1}{2}\right)\left(-\frac{3x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!} \left(-\frac{3x}{4}\right)^2 \dots\right)$$

$$= \frac{1}{2} \left(1 + \frac{3x}{8} + \frac{27x^2}{128}\right)$$

$$= \frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2$$

$$b) \quad \underbrace{(x+8)}_{\substack{= \\ =}} \left(\frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2 \dots \right)$$

$$= \frac{1}{2}x + \frac{3}{16}x^2 + \dots + 4 + \frac{3}{2}x + \frac{27}{32}x^2$$

$$= 4 + 2x + \frac{33}{32}x^2$$



1. $f(x) = \frac{1}{\sqrt{(4+x)}}, \quad |x| < 4$

Find the binomial expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . Give each coefficient as a simplified fraction.

(6)

$$f(x) = (4+x)^{-1/2}$$

$$= 4^{-1/2} \left(1 + \frac{x}{4}\right)^{-1/2}$$

$$= \frac{1}{2} \left(1 + \frac{(-1/2)(-3/2)}{2} \left(\frac{x}{4}\right)^2 + \frac{(-1/2)(-3/2)(-5/2)}{3!} \left(\frac{x}{4}\right)^3\right)$$

$$= \frac{1}{2} - \frac{x}{16} + \frac{3x^2}{256} - \frac{5x^3}{2048}$$



$$5. \quad \frac{2x^2 + 5x - 10}{(x-1)(x+2)} \equiv A + \frac{B}{x-1} + \frac{C}{x+2}.$$

(a) Find the values of the constants A , B and C .

(4)

(b) Hence, or otherwise, expand $\frac{2x^2 + 5x - 10}{(x-1)(x+2)}$ in ascending powers of x , as far as the term in x^2 .

Give each coefficient as a simplified fraction.

(7)

$$\text{Q}) \quad \frac{2x^2 + 5x - 10}{x^2 + x - 2}$$

$$\Rightarrow x^2 + x - 2 \overline{)2x^2 + 5x - 10}$$

$$\begin{array}{r} 2 \\ - 2x^2 - 2x - 4 \\ \hline 3x - 6 \end{array}$$

$$\Rightarrow 2 + \frac{3x - 6}{x^2 + x - 2} \equiv 2 + \frac{3x - 6}{(x-1)(x+2)}$$

$$\Rightarrow 2 + \frac{B}{x-1} + \frac{C}{x+2} \equiv 2 + \frac{B(x+2) + C(x-1)}{(x-1)(x+2)}$$

$$\Rightarrow 3x - 6 \equiv B(x+2) + C(x-1)$$

when $x = 1$

$$-3 = 3B$$

$$\underline{B = -1}$$

when $x = -2$ $-12 = -3C$

$$\underline{C = 4}$$

$$\Rightarrow 2 - \frac{1}{x-1} + \frac{4}{x+2}$$

$$\begin{aligned}
 \text{Q3(b)} \quad & \frac{2x^2 + 5x - 1}{(x-1)(x+2)} = 2 - \frac{1}{(x-1)} + \frac{4}{(x+2)} \\
 &= 2 + \frac{1}{(-x)} + \frac{4}{(2+x)} \\
 &= 2 + (-x)^{-1} + 4(2+x)^{-1} \\
 &= 2 + (-x)^{-1} + 4(2)^{-1} \left(1 + \frac{x}{2}\right)^{-1} \\
 &\quad \swarrow \qquad \downarrow \\
 &= 2 + \left[1 + (-1)(-x) + \frac{(-1)(-2)(-x)^2}{2!} \dots + 2 \left(1 + (-1)\binom{x}{1} + \frac{(-1)(-2)\binom{x}{2}}{2!}\right) \right] \\
 &= 2 + \left[1 + x + x^2 \dots + 2^{-x} + \frac{x^2}{2} \right] \\
 &= 5 + 5x + \frac{3x^2}{2} \\
 &= 5 + \frac{3x^2}{2}
 \end{aligned}$$

2.

$$f(x) = \frac{1}{\sqrt{(9+4x^2)}}, \quad |x| < \frac{3}{2}$$

Find the first three non-zero terms of the binomial expansion of $f(x)$ in ascending powers of x . Give each coefficient as a simplified fraction.

(6)

$$f(x) = (9 + 4x^2)^{-\frac{1}{2}}$$

$$= 9 \left(1 + \frac{4x^2}{9} \right)^{-\frac{1}{2}}$$

$$= \frac{1}{3} \left[1 + \left(-\frac{1}{2} \right) \left(\frac{4}{9} x^2 \right) + \underbrace{\left(-\frac{1}{2} \right) \left(-\frac{3}{2} \right) \left(\frac{4}{9} x^2 \right)^2}_{2} \dots \right]$$

$$= \frac{1}{3} \left[1 - \frac{2}{9} x^2 + \frac{2}{27} x^4 \right]$$

$$= \frac{1}{3} - \frac{2}{27} x^2 + \frac{2}{81} x^4$$



3.

$$f(x) = \frac{6}{\sqrt{(9 - 4x)}}, \quad |x| < \frac{9}{4}$$

- (a) Find the binomial expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . Give each coefficient in its simplest form.

(6)

Use your answer to part (a) to find the binomial expansion in ascending powers of x , up to and including the term in x^3 , of

$$(b) \quad g(x) = \frac{6}{\sqrt{(9 + 4x)}}, \quad |x| < \frac{9}{4} \quad (1)$$

$$(c) \quad h(x) = \frac{6}{\sqrt{(9 - 8x)}}, \quad |x| < \frac{9}{8} \quad (2)$$

$$\text{a) } f(x) = 6(9 - 4x)^{-\frac{1}{2}}$$

$$= 6(9)^{-\frac{1}{2}} \left(1 - \frac{4}{9}x\right)^{-\frac{1}{2}}$$

$$= 2 \left(1 - \frac{4}{9}x\right)^{-\frac{1}{2}}$$

$$= 2 \left[1 + \left(-\frac{1}{2}\right)(-\frac{4}{9}x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2} \left(-\frac{4}{9}x\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{2 \times 3} \left(-\frac{4}{9}x\right)^3 \right]$$

$$= 2 + \frac{4x}{9} + \frac{4x^2}{27} + \frac{40x^3}{729}$$

$$(b) \quad g(x) = 2 - \frac{4x}{9} + \frac{4x^2}{27} - \frac{40x^3}{729} \quad (\text{odd } x \text{ powers become negative})$$

$$(c) \quad h(x) = 2 + \frac{8x}{9} + \frac{16x^2}{27} + \frac{320x^3}{729}$$

$$\qquad \qquad \qquad \begin{matrix} \uparrow \\ x^2 \end{matrix} \qquad \begin{matrix} \uparrow \\ x^2 \end{matrix} \qquad \begin{matrix} \uparrow \\ x^3 \end{matrix}$$



1. Given

$$f(x) = (2 + 3x)^{-3}, \quad |x| < \frac{2}{3}$$

find the binomial expansion of $f(x)$, in ascending powers of x , up to and including the term in x^3 .

Give each coefficient as a simplified fraction.

(5)

$$\begin{aligned}
 (2+3x)^{-3} &= 2^{-3} \left(1 + \frac{3}{2}x\right)^{-3} \\
 &= 2^{-3} \left[1 + \frac{(-3)}{1} \left(\frac{3}{2}x\right) + \frac{(-3)(-4)}{1 \times 2} \left(\frac{3}{2}x\right)^2 \right. \\
 &\quad \left. + \frac{(-3)(-4)(-5)}{1 \times 2 \times 3} \left(\frac{3}{2}x\right)^3 + \dots \right] \\
 &= \frac{1}{8} \left[1 - \frac{9}{2}x + \frac{27}{2}x^2 - \frac{135}{4}x^3 + \dots \right] \\
 &= \frac{1}{8} - \frac{9}{16}x + \frac{27}{16}x^2 - \frac{135}{32}x^3 + \dots
 \end{aligned}$$



2. (a) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \quad |x| < 1 \quad (6)$$

- (b) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

to obtain an approximation to $\sqrt{3}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

$$\sqrt{\frac{1+x}{1-x}} = (1+x)^{\frac{1}{2}} \times (1-x)^{-\frac{1}{2}} \quad (3)$$

$$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}(x) + \frac{\left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{1 \times 2} (x)^2 + \dots$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots$$

$$(1-x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1 \times 2} (-x)^2 + \dots$$

$$= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \dots$$

$$(1+x)^{\frac{1}{2}} \times (1-x)^{-\frac{1}{2}}$$

$$= (1 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots) (1 + \frac{1}{2}x + \frac{3}{8}x^2 + \dots)$$

$$= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \dots$$

$$+ \frac{1}{2}x + \frac{1}{4}x^2 + \dots$$

$$- \frac{1}{8}x + \dots$$

$$= 1 + x + \frac{1}{2}x^2 + \dots$$

as required



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25)
$$\sqrt{\frac{1+\frac{1}{26}}{1-\frac{1}{26}}} = \sqrt{\frac{\frac{27}{26}}{\frac{25}{26}}} = \sqrt{\frac{27}{25}} = \frac{\sqrt{9} \cdot \sqrt{3}}{5}$$
$$= \frac{3\sqrt{3}}{5}$$

$\therefore \frac{3\sqrt{3}}{5} = 1 + \frac{1}{26} + \frac{1}{2} \left(\frac{1}{26}\right)^2$

$$\frac{3\sqrt{3}}{5} = \frac{1405}{1352}$$

$$\sqrt{3} = \frac{1405 \times 5}{1352 \times 3} = \frac{7025}{4056}$$

in form $\frac{a}{b}$ where $a = 7025$
 $b = 4056$