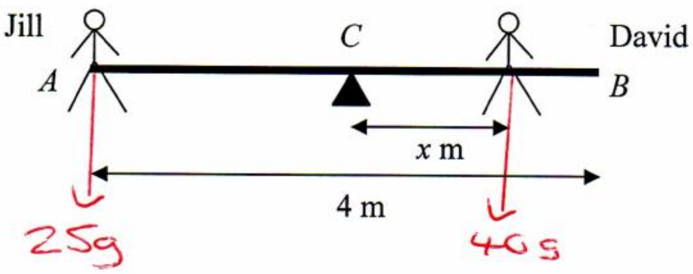
Jan 2006

3.

Figure 1



A seesaw in a playground consists of a beam *AB* of length 4 m which is supported by a smooth pivot at its centre *C*. Jill has mass 25 kg and sits on the end *A*. David has mass 40 kg and sits at a distance *x* metres from *C*, as shown in Figure 1. The beam is initially modelled as a uniform rod. Using this model,

(*a*) find the value of *x* for which the seesaw can rest in equilibrium in a horizontal position.
(3)

(*b*) State what is implied by the modelling assumption that the beam is uniform.
(1)

David realises that the beam is not uniform as he finds he must sit at a distance 1.4 m from *C* for the seesaw to rest horizontally in equilibrium. The beam is now modelled as a non-uniform rod of mass 15 kg. Using this model,

(*c*) find the distance of the centre of mass of the beam from *C*.
(4)

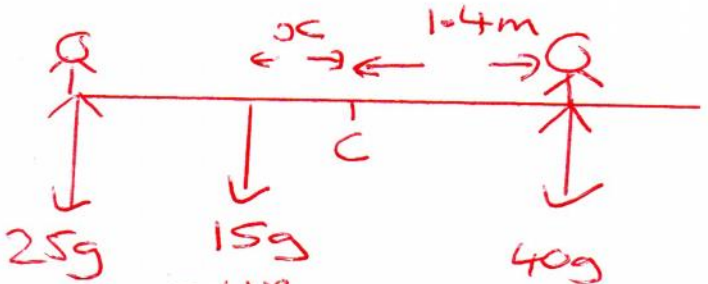
a) M(C) +ve $\quad -2\times 25g + x\times 40g = 0$

$$x \times 40\cancel{g} = 50\cancel{g}$$

$$x = \frac{50}{40} = 1.25\text{m}$$

b) Weight/mass acts at the mid point

c)



M(C) +ve

$$-1.4\times 40g + x\times 15g + 2\times 25g = 0$$

$$x \times 15g = 1.4\times 40g - 50g$$

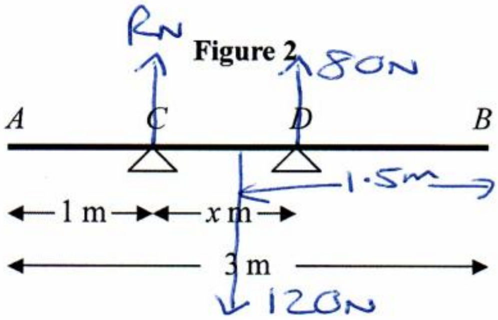
$$x = \frac{1.4\times 40g - 50g}{15g}$$

$x = 0.4$m from C

Leave blank

2.

Figure 2



A uniform plank AB has weight 120 N and length 3 m. The plank rests horizontally in equilibrium on two smooth supports C and D, where $AC = 1$ m and $CD = x$ m, as shown in Figure 2. The reaction of the support on the plank at D has magnitude 80 N. Modelling the plank as a rod,

(a) show that $x = 0.75$

(3)

A rock is now placed at B and the plank is on the point of tilting about D. Modelling the rock as a particle, find

(b) the weight of the rock,

(4)

(c) the magnitude of the reaction of the support on the plank at D.

(2)

(d) State how you have used the model of the rock as a particle.

(1)

a) We do not know force R, so take moments about point C

$M(C) \circlearrowleft^{+}$ anticlockwise

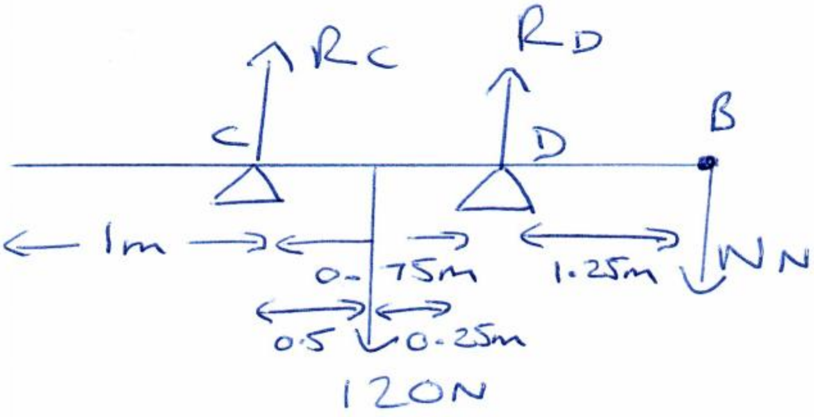
$-0.5 \times 120 + x \times 80 = 0$

$-60 + 80x = 0$

$80x = 60$

$x = \frac{60}{80} = \frac{3}{4} = 0.75\text{ m}$

2b)



If plank is on the point of tilting at D, $R_C = 0$ at point C

We do not know R_D, so take moments about D

$M(D) \curvearrowright^{+}$

$$0.75 \times 0 - 0.25 \times 120 + 1.25W = 0$$

$$0 - 30 + 1.25W = 0$$

$$1.25W = 30$$

$$W = \frac{30}{1.25} = 24\text{N}$$

c) $M(B) \curvearrowright^{+}$

$$1.25 \times R_D - 1.5 \times 120 + 2 \times 0 = 0$$

$$1.25 R_D - 180 = 0$$

$$1.25 R_D = 180$$

$$R_D = \frac{180}{1.25}$$

$$R_D = 144\text{N}$$

d) The weight of the rock acts precisely at B

5.

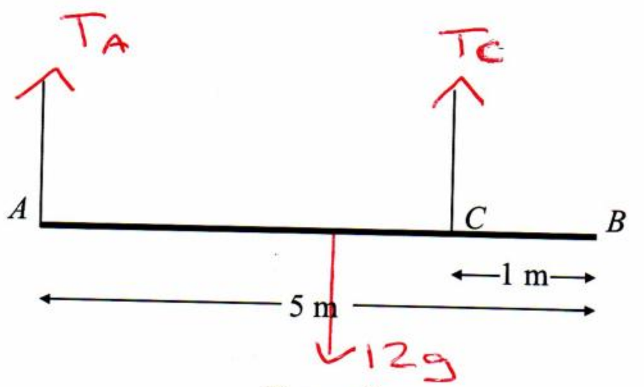


Figure 2

A beam AB has mass 12 kg and length 5 m. It is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A, the other to the point C on the beam, where $BC = 1$ m, as shown in Figure 2. The beam is modelled as a uniform rod, and the ropes as light strings.

(a) Find

(i) the tension in the rope at C,

(ii) the tension in the rope at A.

(5)

A small load of mass 16 kg is attached to the beam at a point which is y metres from A. The load is modelled as a particle. Given that the beam remains in equilibrium in a horizontal position,

(b) find, in terms of y, an expression for the tension in the rope at C.

(3)

The rope at C will break if its tension exceeds 98 N. The rope at A cannot break.

(c) Find the range of possible positions on the beam where the load can be attached without the rope at C breaking.

(3)

a)(i) M(A) ↻ +ve

$$-2.5 \times 12g + 4 \times T_C = 0$$

$$T_C = \frac{2.5 \times 12 \times 9.8}{4}$$

$$T_C = 73.5\text{N}$$

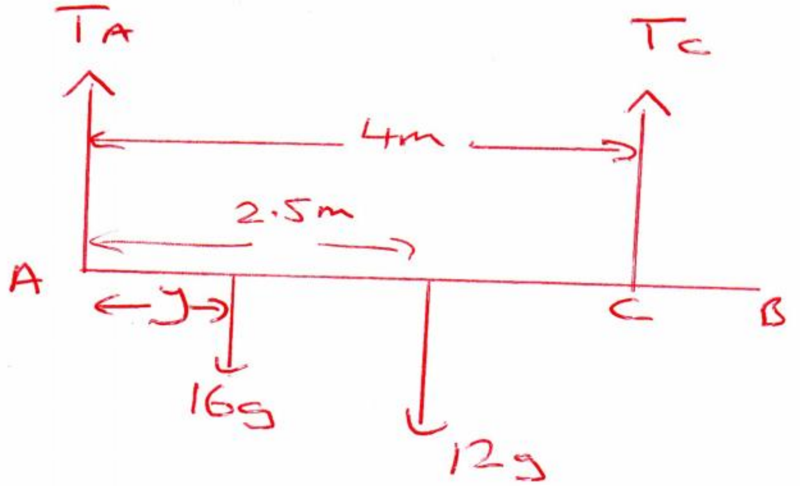
(ii) R (↑) $T_A + T_C - 12g = 0$

$$T_A = (12 \times 9.8) - 73.5$$

$$T_A = 44.1\text{ N}$$

Leave blank

5b)



M(A) ↺ +ve

$$y \times 16g + 2.5 \times 12g - 4 \times T_C = 0$$

$$16gy + 30g = 4T_C$$

$$4gy + 7.5g = T_C$$

$$39.2y + 73.5 = T_C$$

c) If $39.2y + 73.5 = 98$

$$y = \frac{98 - 73.5}{39.2}$$

$$y = 0.625 \text{ m}$$

If tension exceeds 98N the rope will break

This will occur for $y > 0.625$m

So load must be no more than 0.625m from A.

Leave blank

4.

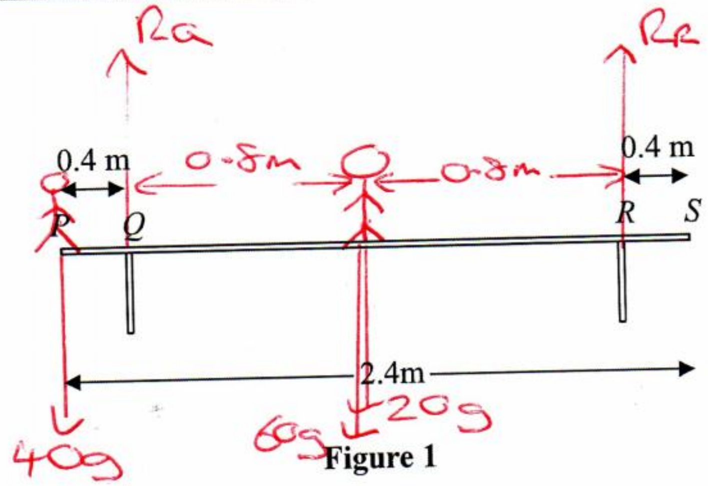


Figure 1

A bench consists of a plank which is resting in a horizontal position on two thin vertical legs. The plank is modelled as a uniform rod PS of length 2.4 m and mass 20 kg. The legs at Q and R are 0.4 m from each end of the plank, as shown in Figure 1.

Two pupils, Arthur and Beatrice, sit on the plank. Arthur has mass 60 kg and sits at the middle of the plank and Beatrice has mass 40 kg and sits at the end P. The plank remains horizontal and in equilibrium. By modelling the pupils as particles, find

(a) the magnitude of the normal reaction between the plank and the leg at Q and the magnitude of the normal reaction between the plank and the leg at R. **(7)**

Beatrice stays sitting at P but Arthur now moves and sits on the plank at the point X. Given that the plank remains horizontal and in equilibrium, and that the magnitude of the normal reaction between the plank and the leg at Q is now twice the magnitude of the normal reaction between the plank and the leg at R,

(b) find the distance QX. **(6)**

a) $R(\uparrow)$ $R_Q + R_R = 40g + 60g + 20g$

$R_Q + R_R = 120g$ (1)

$M(Q)$ $\circlearrowright$ +ve $0.4 \times 40g - 0.8 \times 80g + 1.6 \times R_R = 0$

$156.8 - 627.2 + 1.6 R_R = 0$

$R_R = \dfrac{627.2 - 156.8}{1.6} = 294\text{ N}$

Using R_R in (1)

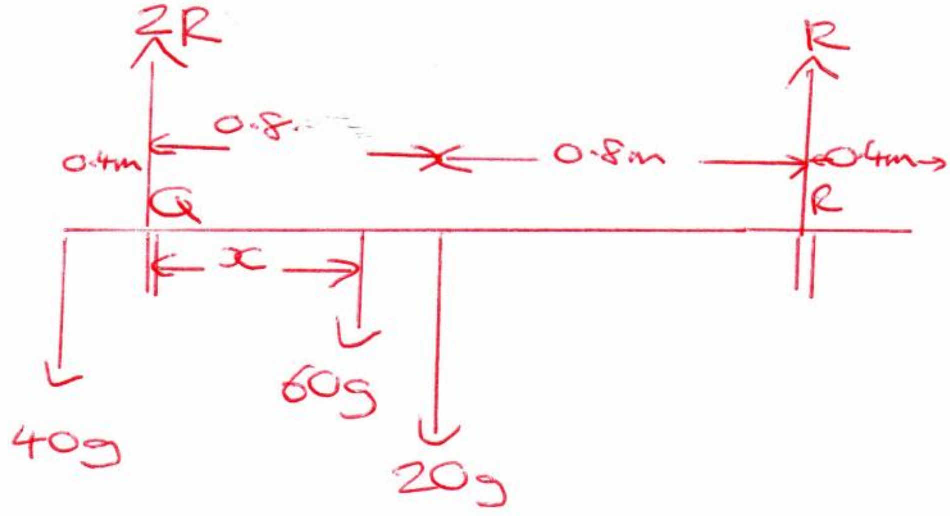
$R_Q = 120 \times 9.8 - 294$

$R_Q = 882\text{ N}$

Normal reaction at R is 294 N

Normal reaction at Q is 882 N

4b)



R (↑)

$$2R + R = 40g + 60g + 20g$$

$$3R = 120g$$

$$R = \frac{120 \times 9.8}{3} = 392N \quad 1$$

M (Q) ↻ +ve

$$-0.4 \times 40g + x \times 60g + (0.8) \times 20g - 1.6 \times R = 0$$

$$\cancel{-156.8} + 588x + \cancel{156.8} - 1.6 \times 392 = 0$$

$$x = \frac{1.6 \times 392}{588}$$

$$x = 1.066666$$

Distance QX $= x = 1.07\,m$ (3sf)

Leave blank

4.

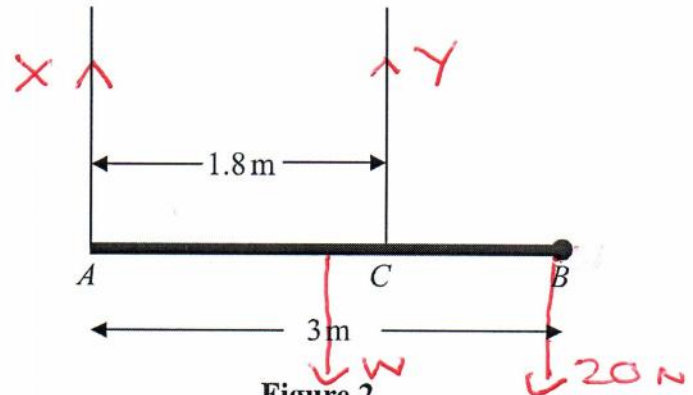


Figure 2

A pole AB has length 3 m and weight W newtons. The pole is held in a horizontal position in equilibrium by two vertical ropes attached to the pole at the points A and C where $AC = 1.8$ m, as shown in Figure 2. A load of weight 20 N is attached to the rod at B. The pole is modelled as a uniform rod, the ropes as light inextensible strings and the load as a particle.

(a) Show that the tension in the rope attached to the pole at C is $\left(\frac{5}{6}W + \frac{100}{3}\right)$ N.

(4)

(b) Find, in terms of W, the tension in the rope attached to the pole at A.

(3)

Given that the tension in the rope attached to the pole at C is eight times the tension in the rope attached to the pole at A,

(c) find the value of W.

(3)

a) M (A) ↻ +ve

$(-1.5 \times W) + (1.8 \times Y) - (3 \times 20) = 0$

$-\frac{3}{2}W + \frac{18}{10}Y - 60 = 0$

(x through by $\frac{10}{18}$)

$-\frac{5}{6}W + Y - \frac{100}{3} = 0$

$Y = \left(\frac{5}{6}W + \frac{100}{3}\right)$ N

as required

b) R (↑)

$X + Y = W + 20$

$X = W + 20 - Y$

$X = W + 20 - \left(\frac{5}{6}W + \frac{100}{3}\right)$

4b) continued

$$X = \left(\frac{1}{6}W - \frac{40}{3}\right) \text{ N}$$

Given $Y = 8X$

c) $$\frac{5}{6}W + \frac{100}{3} = 8\left(\frac{1}{6}W - \frac{40}{3}\right)$$

$$\frac{5}{6}W + \frac{100}{3} = \frac{8}{6}W - \frac{320}{3}$$

(× through by 6)

$$5W + 200 = 8W - 640$$

$$840 = 3W$$

$$W = 280 \text{ N}$$

M1 JAN 2011

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3.

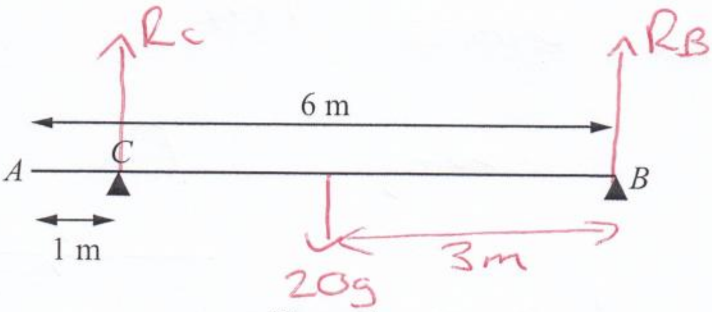


Figure 1

A uniform beam AB has mass 20 kg and length 6 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at C, where $AC = 1$ m, and the other is at the end B, as shown in Figure 1. The beam is modelled as a rod.

(a) Find the magnitudes of the reactions on the beam at B and at C.

(5)

A boy of mass 30 kg stands on the beam at the point D. The beam remains in equilibrium. The magnitudes of the reactions on the beam at B and at C are now equal. The boy is modelled as a particle.

(b) Find the distance AD.

(5)

a) R (↑)

$R_C + R_B = 20g$ (1)

M(B) (↻ +ve)

$-3\times 20g + 5\times R_C = 0$ (2)

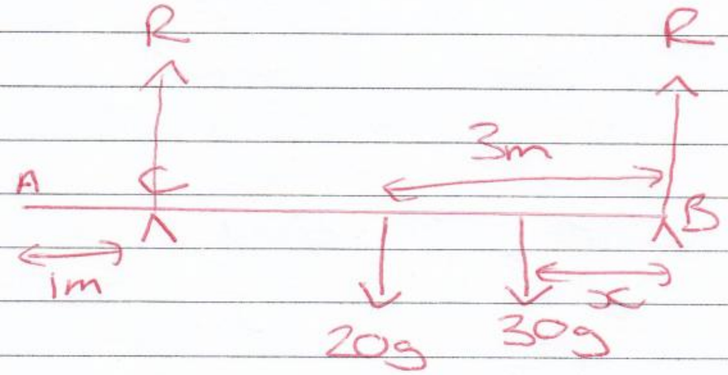
$R_C = \frac{60\times 9.8}{5}$

$R_C = 117.6\text{ N}$

in (1) $R_B = (20\times 9.8) - 117.6$

$R_B = 78.4\text{ N}$

b)



M1 JAN 2011

3b) R (↑)

$$R + R = 20g + 30g$$
$$2R = 50g$$
$$R = 25 \times 9 \cdot 8$$
$$R = 245N$$

$M(B) \circlearrowleft^{+ve}$

$$x \times 30g + 3 \times 20g - 5 \times 245 = 0$$
$$x = \frac{(5 \times 245) - (60 \times 9 \cdot 8)}{30 \times 9 \cdot 8}$$
$$x = 2\tfrac{1}{6}m$$

Distance AD $= 6 - 2\frac{1}{6}$
$= 3\frac{5}{6}m$

M1 Jan 2012

4.

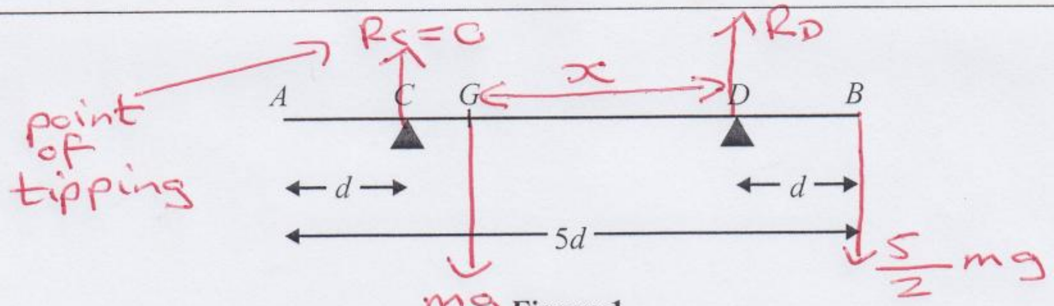


Figure 1

A non-uniform rod AB, of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D, where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G. A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D.

(a) Show that $GD = \frac{5}{2}d$.

(4)

The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at D and the rod.

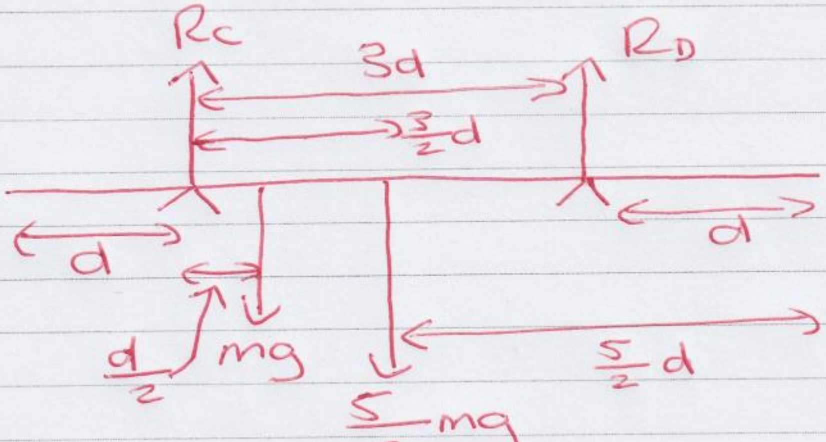
(5)

a) $m(D)\ \circlearrowleft^{+ve}$ to eliminate R_D

$$x \times \cancel{mg} - d \times \frac{5}{2}\cancel{mg} = 0$$

$$GD = x = \frac{5}{2}d \quad \text{as required}$$

b)



$m(C)\ \circlearrowleft^{+ve}$ $-\frac{d}{2} \times mg - \frac{3}{2} \times \frac{5}{2}mg + 3d \times R_D = 0$

$$-\frac{d}{2}mg - \frac{15d}{4}mg + 3d\,R_D = 0$$

$$3\cancel{d}\,R_D = \frac{15\cancel{d}}{4}mg + \frac{2\cancel{d}}{4}mg$$

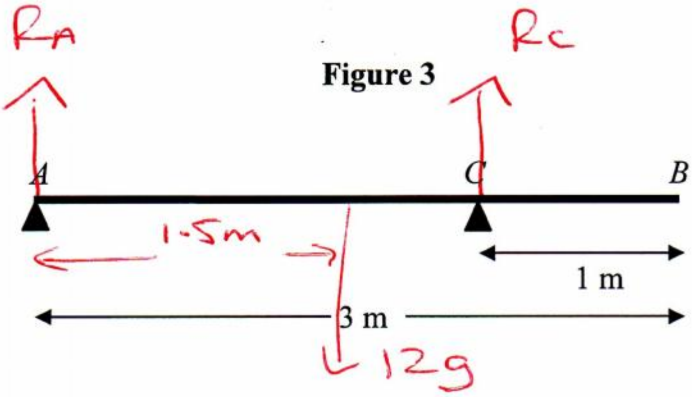
$$3R_D = \frac{17}{4}mg$$

Normal Reaction at D is $R_D = \frac{17}{12}mg$

June 2005

6.

Figure 3



A uniform beam AB has mass 12 kg and length 3 m. The beam rests in equilibrium in a horizontal position, resting on two smooth supports. One support is at end A, the other at a point C on the beam, where $BC = 1$ m, as shown in Figure 3. The beam is modelled as a uniform rod.

(*a*) Find the reaction on the beam at C.

(3)

A woman of mass 48 kg stands on the beam at the point D. The beam remains in equilibrium. The reactions on the beam at A and C are now equal.

(*b*) Find the distance AD.

(7)

---

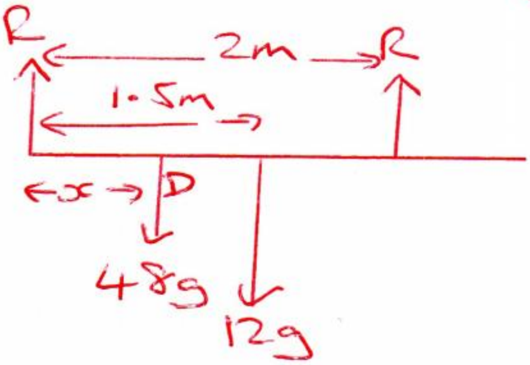
a) M(A) ↺ +ve $\quad -1.5 \times 12g + 2 \times R_c = 0$

$$2R_c = 1.5 \times 12 \times 9.8$$

$$R_c = \frac{1.5 \times 12 \times 9.8}{2}$$

$$R_c = 88.2\,N$$

b)



R(↑)

$$R + R - 48g - 12g = 0$$

$$2R = 60g$$

$$R = 30g$$

M(A) ↻ +ve $\quad x \times 48g + 1.5 \times 12g - 2 \times 30g = 0$

$$x \times 48g = 2 \times 30g - 1.5 \times 12g$$

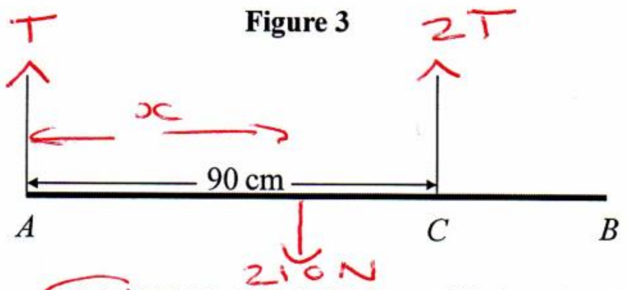
$$x = \frac{2 \times 30g - 1.5 \times 12g}{48g}$$

Distance AD $= x = 0.875\,m$

June 2006

5.

Figure 3



A steel girder AB has weight 210 N. It is held in equilibrium in a horizontal position by two vertical cables. One cable is attached to the end A. The other cable is attached to the point C on the girder, where $AC = 90$ cm, as shown in Figure 3. The girder is modelled as a uniform rod, and the cables as light inextensible strings.

Given that the tension in the cable at C is twice the tension in the cable at A, find

(a) the tension in the cable at A,

(2)

(b) show that $AB = 120$ cm.

(4)

A small load of weight W newtons is attached to the girder at B. The load is modelled as a particle. The girder remains in equilibrium in a horizontal position. The tension in the cable at C is now three times the tension in the cable at A.

(c) Find the value of W.

(7)

a) $R(\uparrow)$ $T + 2T - 210 = 0$

$3T = 210$

$T = 70$ N

Tension at A is 70N

b) Let x be distance to Centre of mass

M (A) ↻ +ve

$x \times 210 - 0.9 \times 2 \times 70 = 0$

$x = \dfrac{0.9 \times 2 \times 70}{210}$

$x = 0.6$m

As this is distance to centre,

$AB = 2 \times 0.6 = 1.2$m $= 120$ cm

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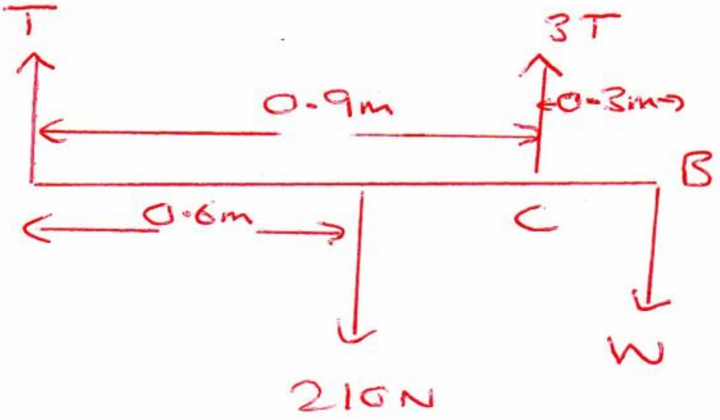
5c)



$R(\uparrow)\quad T + 3T - 210 - W = 0$

$4T = 210 + W \quad (1)$

To eliminate W take moments at B

$M(B) \circlearrowright +ve \quad 0.3 \times 3T - 0.6 \times 210 + 1.2 \times T = 0$

$0.9T + 1.2T = 0.6 \times 210$

$2.1T = 126$

$T = \dfrac{126}{2.1}$

$T = 60\,N$

Put $T = 60N$ in (1)

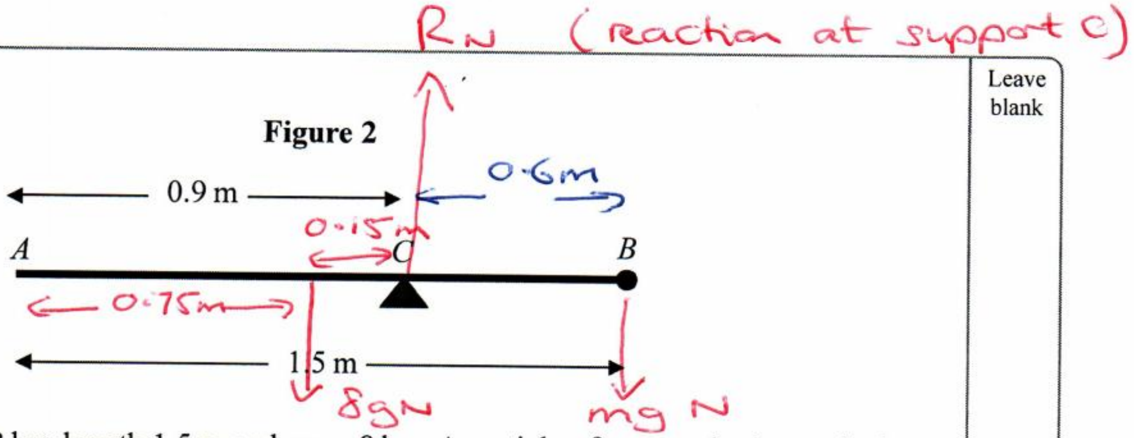
$4 \times 60 - 210 = W$

$W = 240 - 210$

$W = 30\,N$

3.

Figure 2



A uniform rod AB has length 1.5 m and mass 8 kg. A particle of mass m kg is attached to the rod at B. The rod is supported at the point C, where $AC = 0.9$ m, and the system is in equilibrium with AB horizontal, as shown in Figure 2.

(a) Show that $m = 2$.

(4)

A particle of mass 5 kg is now attached to the rod at A and the support is moved from C to a point D of the rod. The system, including both particles, is again in equilibrium with AB horizontal.

(b) Find the distance AD.

(5)

if we take "Moments about C" we take R out of calculation

$M(C)\circlearrowright_{+}$ – taking moments about C in clockwise direction to keep mass "m" +ve

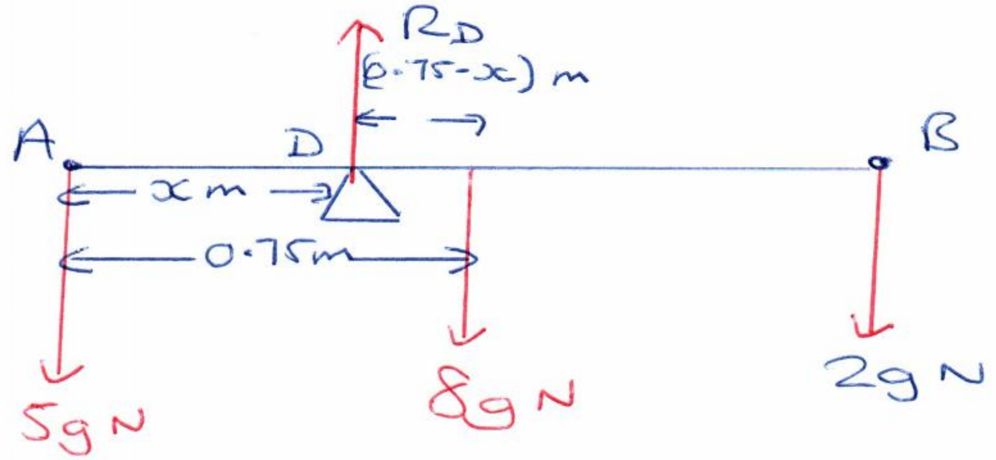
$mg(0.6) - 8g(0.15) = 0$ (in equilibrium

$\therefore\ 0.6m\cancel{g} - 1.2\cancel{g} = 0$

$0.6m = 1.2$

$m = 2$ kg

3b)



Take moments about D to eliminate force R_D, moments anticlockwise to keep x +ve

$M(D)$ $\circlearrowleft +$

$$5\cancel{g}x - 8\cancel{g}(0.75-x) = 0$$
$$- 2\cancel{g}(1.5-x)$$

distance DB

divide through by g

$\therefore\ 5x - 6 + 8x - 3 + 2x = 0$

$\therefore\ 15x - 9 = 0$

$15x = 9$

$x = \frac{9}{15} = \frac{3}{5}$ m

$x = 0.6$ m

AD = 0.6m

Leave blank

7.

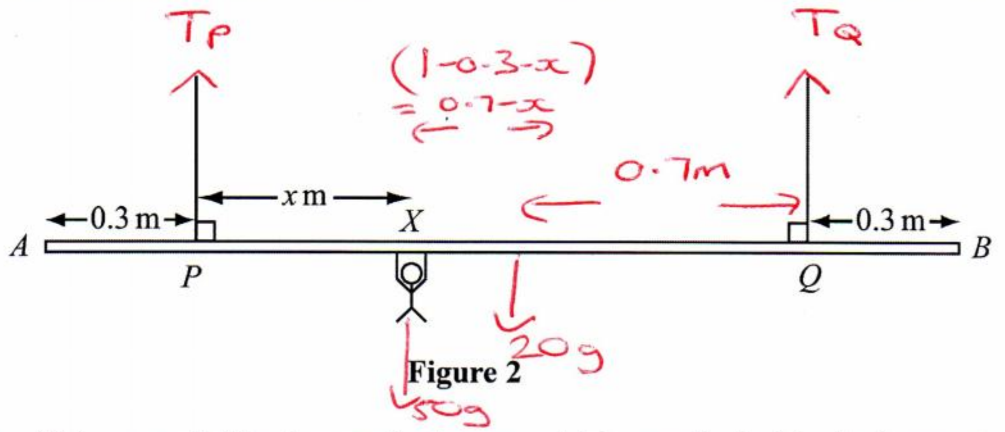


Figure 2

A beam AB is supported by two vertical ropes, which are attached to the beam at points P and Q, where $AP = 0.3$ m and $BQ = 0.3$ m. The beam is modelled as a uniform rod, of length 2 m and mass 20 kg. The ropes are modelled as light inextensible strings. A gymnast of mass 50 kg hangs on the beam between P and Q. The gymnast is modelled as a particle attached to the beam at the point X, where $PX = x$ m, $0 < x < 1.4$ as shown in Figure 2. The beam rests in equilibrium in a horizontal position.

(a) Show that the tension in the rope attached to the beam at P is $(588 - 350x)$ N. **(3)**

(b) Find, in terms of x, the tension in the rope attached to the beam at Q. **(3)**

(c) Hence find, justifying your answer carefully, the range of values of the tension which could occur in each rope. **(3)**

Given that the tension in the rope attached at Q is three times the tension in the rope attached at P,

(d) find the value of x. **(3)**

a) $M(Q)$ $\circlearrowright$ +ve

$-0.7 \times 20g - (0.7 + 0.7 - x) \times 50g + 1.4 \times T_P = 0$

$-137.2 - (1.4 - x) \times 50g + 1.4 \times T_P = 0$

$1.4 T_P = 137.2 + 50g(1.4 - x)$

$T_P = \frac{137.2}{1.4} + \frac{50g}{1.4}(1.4 - x)$

$T_P = 98 + 50g - \frac{50g}{1.4}x$

$T_P = 98 + (50 \times 9.8) - \frac{50 \times 9.8}{1.4}x$

$T_P = (588 - 350x)$ N

as requested

7b) R (↑)

$T_P + T_Q - 50g - 20g = 0$ (1)

$T_Q = 70g - T_P$

$T_Q = 70 \times 9.8 - (588 - 350x)$

$T_Q = 686 - 588 + 350x$

$T_Q = (98 + 350x)$ N

c) Since $0 < x < 1.4$

$T_P = 588 - 350x$

$T_P max$ is < 588

$T_P min > 588 - (350 \times 1.4)$

> 98

so $98 < T_P < 588$

(as x cannot be 0 or 1.4)

$T_Q = 98 + 350x$

$T_P min > 98$

$T_P max < 98 + (350 \times 1.4)$

< 588

so $98 < T_Q < 588$

d) As $T_Q = 3T_P$

$(98 + 350x) = 3(588 - 350x)$

$98 + 350x = 1764 - 1050x$

$1050x + 350x = 1764 - 98$

$1400x = 1666$

$x = \frac{1666}{1400}$

$\underline{\underline{x = 1.19 \text{ m}}}$

6.

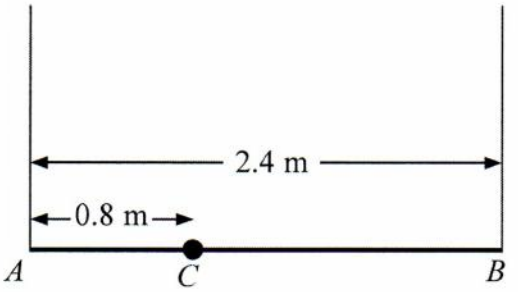


Figure 2

A plank AB has mass 12 kg and length 2.4 m. A load of mass 8 kg is attached to the plank at the point C, where $AC = 0.8$ m. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes, one attached at A and the other attached at B, as shown in Figure 2. The plank is modelled as a uniform rod, the load as a particle and the ropes as light inextensible strings.

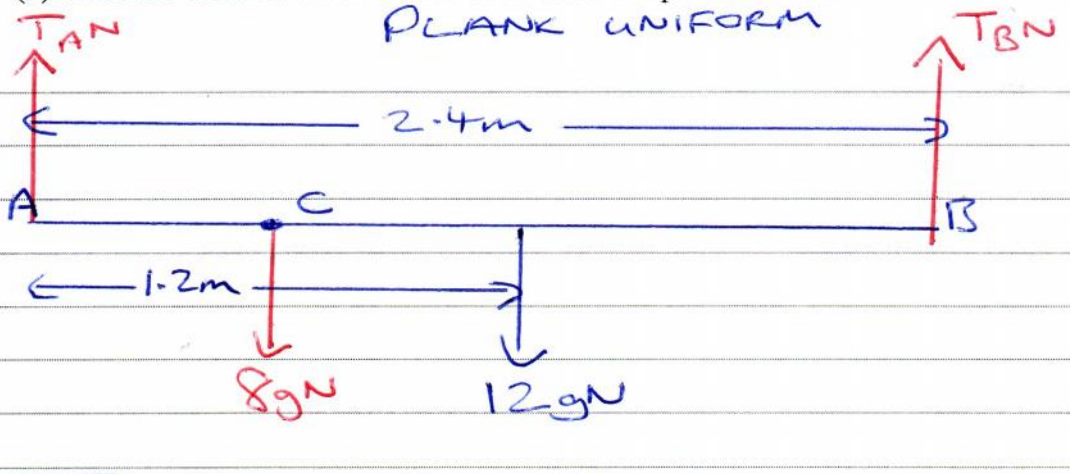
Tensions T_A and T_B will be different

(a) Find the tension in the rope attached at B. **(4)**

The plank is now modelled as a non-uniform rod. With the new model, the tension in the rope attached at A is 10 N greater than the tension in the rope attached at B.

(b) Find the distance of the centre of mass of the plank from A. **(6)**

PLANK UNIFORM



a) Take moments about A in an anticlockwise direction (want T_B to be positive)

$M(A)\circlearrowleft^{+}$

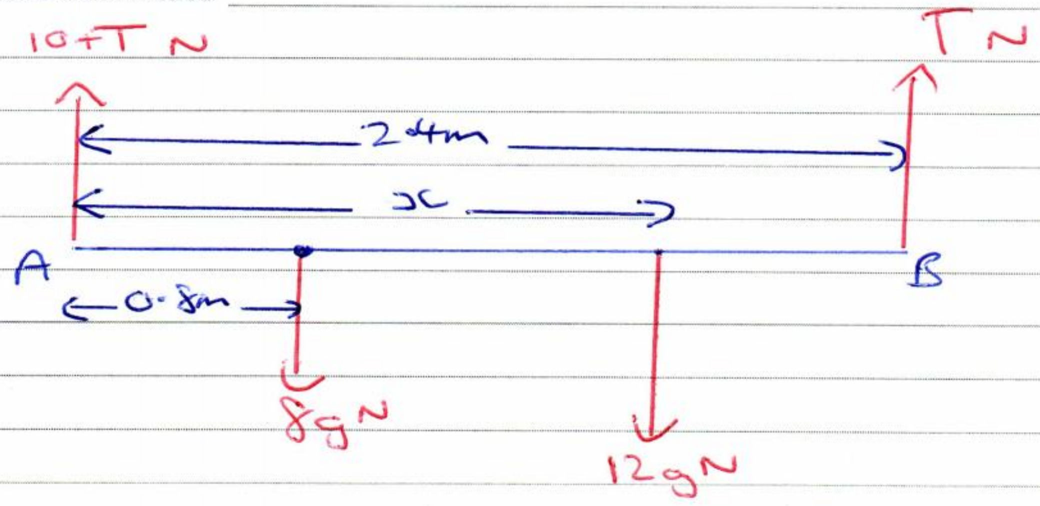
$$T_B(2.4) - 12g(1.2) - 8g(0.8) = 0$$
(rod in equilibrium)

$$\therefore 2.4T_B = 203.84$$

$$T_B = \frac{203.84}{2.4} = 84.9\dot{3}$$
$$= 84.9\text{N (3sf)}$$

Question 6 continued

Plank no longer uniform



Plank in equilibrium

b) R (↑)

$10 + T + T - 8g - 12g = 0$

$\therefore 2T = 20g - 10$

$\therefore T = 10g - 5$ N

Then take moments about A clockwise (keep $x \times 12g$ positive)

M (A) +

$T = 10g - 5$

$12gx + 8g(0.8) - 2.4(10g - 5) = 0$

$\therefore 12gx + 62.72 - 223.2 = 0$

$\therefore 12gx = 223.2 - 62.72$

$\therefore x = \dfrac{223.2 - 62.72}{12g}$

$\therefore x = 1.364...$

$= 1.4$ m (1dp)

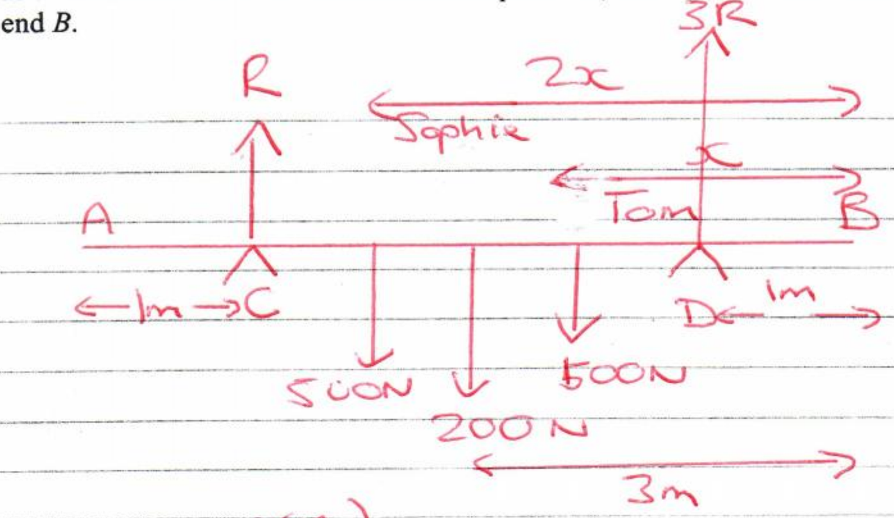
Leave blank

May 2010

Leave blank

4. A beam AB has length 6 m and weight 200 N. The beam rests in a horizontal position on two supports at the points C and D, where $AC = 1$ m and $DB = 1$ m. Two children, Sophie and Tom, each of weight 500 N, stand on the beam with Sophie standing twice as far from the end B as Tom. The beam remains horizontal and in equilibrium and the magnitude of the reaction at D is three times the magnitude of the reaction at C. By modelling the beam as a uniform rod and the two children as particles, find how far Tom is standing from the end B.

(7)



Resolve (↑)

$$R + 3R = 500 + 200 + 500$$
$$4R = 1200$$
$$R = 300\text{ N}$$

M (B) ↺ +ve

$$(-1 \times 3R) + (x \times 500) + (2x \times 500) + (3 \times 200) - (5 \times R) = 0$$

$$(-3 \times 300) + 500x + 1000x + 600 - (5 \times 300) = 0$$

$$-900 + 1500x + 600 - 1500 = 0$$
$$1500x = 1500 - 600 + 900$$
$$1500x = 1800$$
$$x = 1.2\text{ m}$$

Tom is standing 1.2m from B

May 2011

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5. A plank PQR, of length 8 m and mass 20 kg, is in equilibrium in a horizontal position on two supports at P and Q, where $PQ = 6$ m.

A child of mass 40 kg stands on the plank at a distance of 2 m from P and a block of mass M kg is placed on the plank at the end R. The plank remains horizontal and in equilibrium. The force exerted on the plank by the support at P is equal to the force exerted on the plank by the support at Q.

By modelling the plank as a uniform rod, and the child and the block as particles,

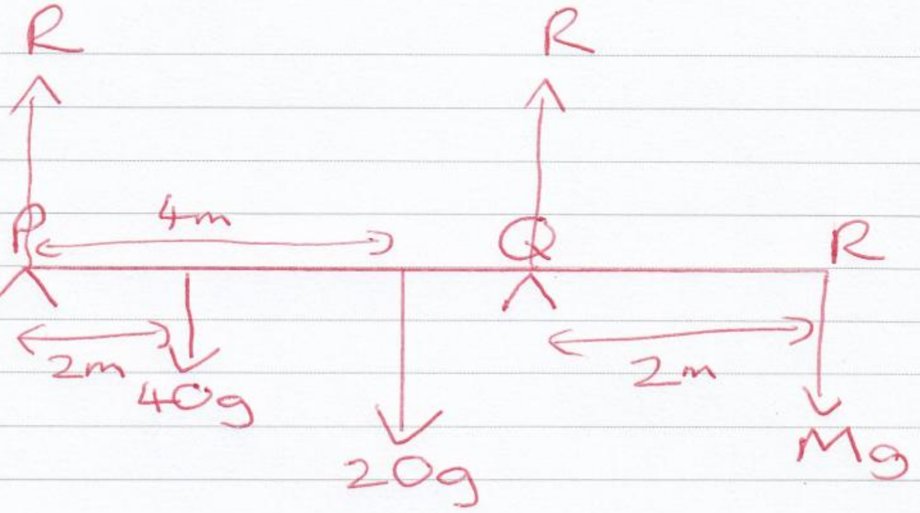
(a) (i) find the magnitude of the force exerted on the plank by the support at P,

(ii) find the value of M.

(10)

(b) State how, in your calculations, you have used the fact that the child and the block can be modelled as particles.

(1)



a) $R(\uparrow)$

$R + R - 40g - 20g - Mg = 0$ (1)

$m(R)$ +ve

$2 \times R - 4 \times 20g - 6 \times 40g + 8 \times R = 0$

$10R = 80g + 240g$

$10R = 320g$

(i) $R = \dfrac{320 \times 9.8}{10} = 313.6\text{N}$

(ii) in (1)

$313.6 + 313.6 - 40g - 20g = Mg$

$M = \dfrac{313.6 + 313.6 - (40 \times 9.8) - (20 \times 9.8)}{9.8}$

$M = 4$

M1 MAY 2012

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2.

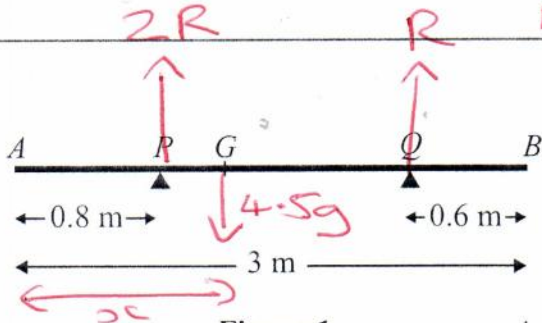

Figure 1

A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q, where $AP = 0.8$ m and $QB = 0.6$ m, as shown in Figure 1. The centre of mass of the rod is at G. Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

(a) the magnitude of the reaction of the support at Q on the rod, **(3)**

(b) the distance AG. **(4)**

a) $R(\uparrow)$

$$2R + R - 4.5g = 0$$
$$3R = 4.5 \times 9.8$$
$$R = \frac{4.5 \times 9.8}{3} = 14.7\text{N}$$

Magnitude of reaction at Q is 14.7N

b) $m(A)$ ↻ +ve

$$-0.8 \times 2R + x \times 4.5g - 2.4 \times R = 0$$
$$-0.8 \times 29.4 + 44.1x - 35.28 = 0$$
$$4.41x = 35.28 + 0.8 \times 29.4$$
$$x = \frac{35.28 + 23.52}{44.1}$$
$$x = 1\tfrac{1}{3}$$

AG is $1\tfrac{1}{3}$m

M1 JAN 2013

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2. A steel girder AB, of mass 200 kg and length 12 m, rests horizontally in equilibrium on two smooth supports at C and at D, where $AC = 2$ m and $DB = 2$ m. A man of mass 80 kg stands on the girder at the point P, where $AP = 4$ m, as shown in Figure 1.

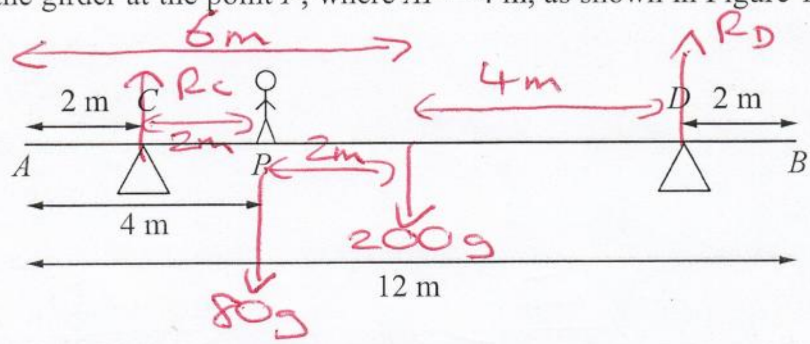


Figure 1

The man is modelled as a particle and the girder is modelled as a uniform rod.

(a) Find the magnitude of the reaction on the girder at the support at C.

(3)

The support at D is now moved to the point X on the girder, where $XB = x$ metres. The man remains on the girder at P, as shown in Figure 2.

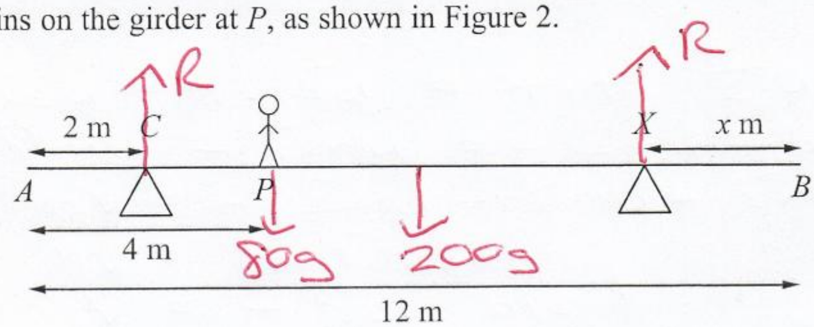


Figure 2

Given that the magnitudes of the reactions at the two supports are now equal and that the girder again rests horizontally in equilibrium, find

(b) the magnitude of the reaction at the support at X,

(2)

(c) the value of x.

(4)

a) M(D) +ve

$-4\times 200g - 6\times 80g + 8\times R_C = 0$

$8R_C = 800g + 480g$

$8R_C = 1280g$

$R_C = 160g = 160\times 9.8$

$= \underline{1568\text{ N}}$

M1 JAN 2013

2b) R (↑) $2R = 280g$

$$R = \frac{280 \times 9.8}{2}$$

$R = 1372N$

Reaction at X is $\underline{\underline{1372N}}$

c) M(B) $\circlearrowright$ +ve

$x \times 1372 - 6 \times 200g - 8 \times 80g$
$+ 10 \times 1372 = 0$

$1372x = 1200g + 640g - 13720$

$1372x = 1200 \times 9.8 + 640 \times 9.8 - 13720$

$1372x = 4312$

$$x = \frac{4312}{1372} = 3.1428571$$

$x = \underline{\underline{3.14 m}}$ (3sf)

M1 MAY 2013

6a) continued

① gives $x m \cancel{g} = 100 \cancel{g}$

$$x = \frac{100}{m}$$

sub in ②

$$m\cancel{g}\left(10 - \frac{100}{m}\right) = 150\cancel{g}$$

$$10m - 100 = 150$$

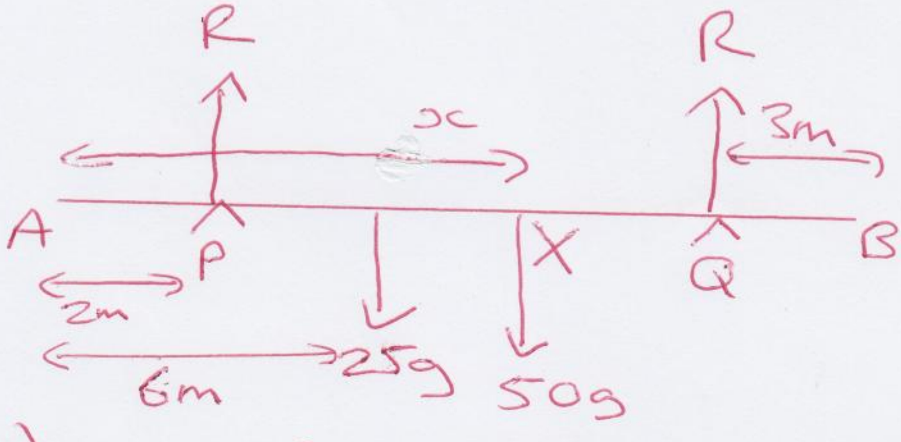
$$10m = 250$$

(i) $\underline{\underline{m = 25 \text{ kg}}}$

(ii) $x = \frac{100}{m} = \frac{100}{25} = 4\text{m}$

Distance of centre of mass from A is $2+4 = \underline{\underline{6\text{m}}}$

b)



R(↑) $R + R = 75g$

$$R = 37{\cdot}5g \text{ N}$$

M(A) ↻+ve $-2 \times 37{\cdot}5g + 6 \times 25g + x \times 50g - 12 \times 37{\cdot}5g = 0$

$$-75g + 150g + 50xg - 450g = 0$$

$$50xg = 450g - 150g + 75g$$

$$x = \frac{375g}{50g} = 7{\cdot}5\text{m}$$

$\underline{\underline{AX = 7{\cdot}5\text{m}}}$

M1 MAY 2013

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6. A beam AB has length 15 m. The beam rests horizontally in equilibrium on two smooth supports at the points P and Q, where $AP = 2$ m and $QB = 3$ m. When a child of mass 50 kg stands on the beam at A, the beam remains in equilibrium and is on the point of tilting about P. When the same child of mass 50 kg stands on the beam at B, the beam remains in equilibrium and is on the point of tilting about Q. The child is modelled as a particle and the beam is modelled as a non-uniform rod.

(a) (i) Find the mass of the beam.

(ii) Find the distance of the centre of mass of the beam from A.

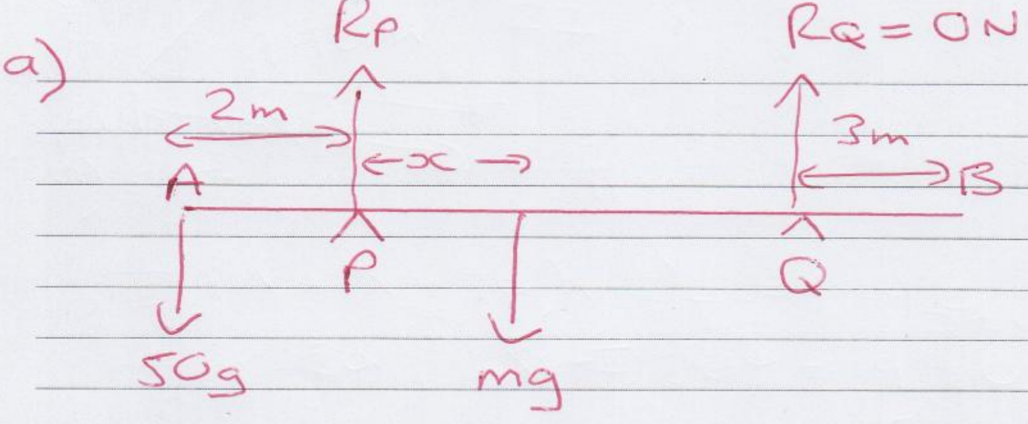
(8)

When the child stands at the point X on the beam, it remains horizontal and in equilibrium. Given that the reactions at the two supports are equal in magnitude,

(b) find AX.

(6)

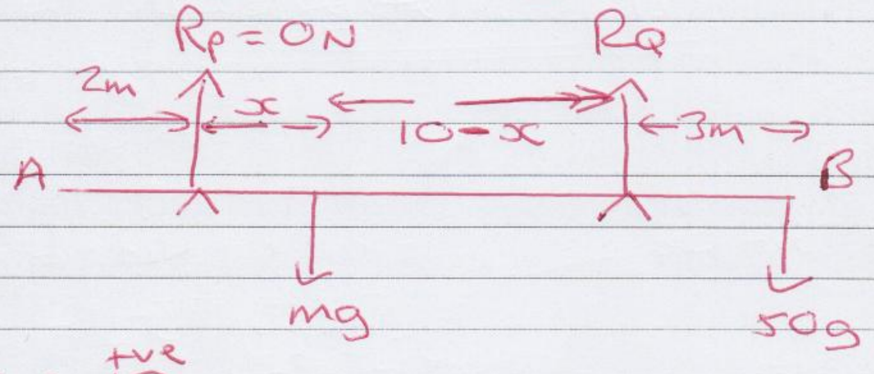
a)



When child 50 kg at A, if tilting at P, $R_Q = 0$ N

$m(P)$ +ve

$x \times mg - 2 \times 50g = 0$ (1)



$m(Q)$ +ve

$(10-x)mg - 3 \times 50g = 0$ (2)