Jan 2006

2. The sequence of positive numbers $u_1, u_2, u_3, \ldots$, is given by

$$u_{n+1} = (u_n - 3)^2, \qquad u_1 = 1.$$

(*a*) Find u_2, u_3 and u_4.

(3)

(*b*) Write down the value of u_{20}.

(1)

a) $u_2 = (u_1 - 3)^2$
$u_2 = (1 - 3)^2 = (-2)^2 = 4$

$u_3 = (u_2 - 3)^2 = (4 - 3)^2 = 1$
$u_4 = (u_3 - 3)^2 = (1 - 3)^2 = (-2)^2 = 4$

b) $u_{20} = 4$
as all even values will be 4

Jan 2006

7. On Alice's 11th birthday she started to receive an annual allowance. The first annual allowance was £500 and on each following birthday the allowance was increased by £200.

(*a*) Show that, immediately after her 12th birthday, the total of the allowances that Alice had received was £1200. **(1)**

(*b*) Find the amount of Alice's annual allowance on her 18th birthday. **(2)**

(*c*) Find the total of the allowances that Alice had received up to and including her 18th birthday. **(3)**

When the total of the allowances that Alice had received reached £32 000 the allowance stopped.

(*d*) Find how old Alice was when she received her last allowance. **(7)**

(n) Term	1st	2nd		8th
Age	11	12	13	18
Allowance	500	700	900	

a) allowances received after 12^{th} birthday

$= 500 + 700 = \underline{\underline{£1200}}$

b) Expression for n^{th} term

Age 11 is 1st term $(n=1)$

$\underbrace{500 \quad 700}_{+200} \quad 900$

$200n + 300$ is n^{th} term

18^{th} allowance is 8^{th} term

$= 200 \times 8 + 300 = 1600 + 300$

$= \underline{\underline{£1900}}$

c) $S_n = \frac{n}{2}(2a + (n-1)d$

$S_8 = \frac{8}{2}[(2 \times 500 + (8-1)(200)]$

$= 4[1000 + (7 \times 200)]$

$= 4[1000 + 1400]$

$= 4 \times 2400 = \underline{\underline{£9600}}$

d) $S_n = \frac{n}{2}[2a + (n-1)d]$

d) continued

$$32000 = \frac{n}{2}\left[(2\times 500)+(n-1)200\right]$$

$$32000 = \frac{n}{2}\left[1000+200n-200\right]$$

$\times$ through by 2

$$64000 = n\left[800+200n\right]$$

$$64000 = 800n+200n^2$$

$$0 = 200n^2+800n-64000$$

$$0 = 200(n^2+4n-320)$$

$$0 = 200(n-16)(n+20)$$

Either $n=16$ or $n=-20$ (impossible)

$$n=16$$

16th term $(n=16)$ represents age 26

Alice is 26 when she receives her last allowance

Jan 2007

9. Ann has some sticks that are all of the same length. She arranges them in squares and has made the following 3 rows of patterns:

$n=0$ | $n=1$ $n=2$ $n=3$

Row 1  1 | 4 $\xrightarrow{}$ 7 $\xrightarrow{}$ 10

Row 2  +3

Row 3  nth term is $3n+1$

She notices that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares.

(a) Find an expression, in terms of n, for the number of sticks required to make a similar arrangement of n squares in the nth row. **(3)**

Ann continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

(b) Find the total number of sticks Ann uses in making these 10 rows. **(3)**

Ann started with 1750 sticks. Given that Ann continues the pattern to complete k rows but does not have sufficient sticks to complete the $(k+1)$th row,

(c) show that k satisfies $(3k-100)(k+35)<0$. **(4)**

(d) Find the value of k. **(2)**

a) $3n+1$

$a=4,\ d=3$

b) $S_{10} = \frac{10}{2}\left((2\times4)+(10-1)(3)\right)$

$= 5(8+(9\times3))$

$= 5(8+27)$

$= 5\times35 = \underline{\underline{175}}$

c) $S_k < 1750$

$\frac{k}{2}\left((2\times4)+(k-1)\times3\right) < 1750$

$\frac{k}{2}(8+3k-3) < 1750$

$\times$ through by 2

$k(5+3k) < 3500$

$5k+3k^2-3500 < 0$

$(3k-100)(k+35) < 0$

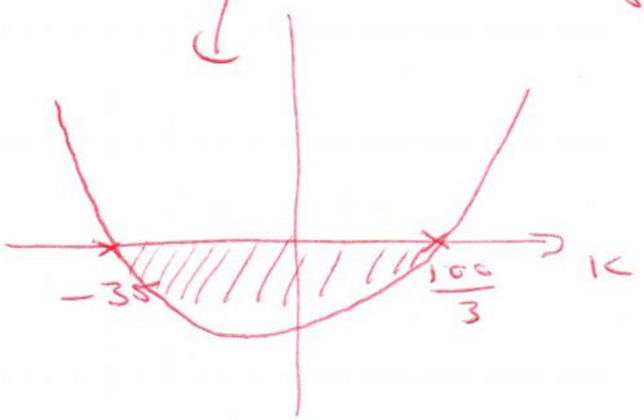
as required

d) Either $3k - 100 = 0$ or $k + 35 = 0$

(critical values)

$k = \frac{100}{3}$ or $k = -35$

From diagram $-35 < k < \frac{100}{3}$



k cannot be $-ve$

so $k < \frac{100}{3}$ $(33\frac{1}{3})$

but as k is an integer

$\underline{\underline{k = 33}}$

as curve

$(3k-100)(k+35) < 0$

$-35 < k < \frac{100}{3}$

Leave blank

7. A sequence is given by:

$$x_1 = 1,$$
$$x_{n+1} = x_n(p + x_n),$$

where p is a constant ($p \neq 0$).

(a) Find x_2 in terms of p.

(1)

(b) Show that $x_3 = 1 + 3p + 2p^2$.

(2)

Given that $x_3 = 1$,

(c) find the value of p,

(3)

(d) write down the value of x_{2008}.

(2)

a) $x_2 = x_1(p + x_1)$
$x_2 = 1(p+1)$
$x_2 = p+1$

b) $x_3 = x_2(p + x_2)$
$x_3 = (p+1)(p+p+1)$
$= (p+1)(2p+1)$
$= 2p^2 + p + 2p + 1$
$= 2p^2 + 3p + 1$
(as required)

c) $1 = 2p^2 + 3p + 1$
$0 = 2p^2 + 3p$
$0 = p(2p+3)$
$p \neq 0$ so $2p + 3 = 0$
$p = -\frac{3}{2}$

d) $x_1 = 1,\ x_2 = -\frac{3}{2} + 1 = -\frac{1}{2},$
$x_3 = 2\left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) + 1$
$x_3 = 2 \times \frac{9}{4} - \frac{9}{2} + 1 = 1$
$x_4 = x_3\left(-\frac{3}{2} + x_3\right) = 1\left(-\frac{3}{2} + 1\right) = -\frac{1}{2}$
All even terms $= -\frac{1}{2}$ $\therefore x_{2008} = -\frac{1}{2}$

Leave blank

11. The first term of an arithmetic sequence is 30 and the common difference is -1.5

(a) Find the value of the 25th term. **(2)**

$a = 30$

The rth term of the sequence is 0.

$d = -1.5$

(b) Find the value of r. **(2)**

The sum of the first n terms of the sequence is S_n.

(c) Find the largest positive value of S_n. **(3)**

a) 25^{th} term $= a + 24d$

$= 30 + 24(-1.5)$

$= 30 - 36$

$= -6$

b) r^{th} term

$\quad\quad\quad\quad a \quad\quad\quad d$

$0 = 30 + (-1.5)(r-1)$

$0 = 30 - 1.5r + 1.5$

$1.5r = 31.5$

$\frac{3}{2}r = 31.5$

$3r = 63$

$r = 21$

c) The 21^{st} term is zero, so terms after this will be negative (as $d = -1.5$)

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{21} = \frac{21}{2}(2 \times 30 + (20)(-1.5))$

$= \frac{21}{2}(60 - 30)$

$= \frac{21}{2} \times 30 = \frac{630}{2} = 315$

Largest positive value of $S_n = 315$

Leave blank

9. The first term of an arithmetic series is a and the common difference is d.

The 18th term of the series is 25 and the 21st term of the series is $32\frac{1}{2}$.

(a) Use this information to write down two equations for a and d.
(2)

(b) Show that $a = -17.5$ and find the value of d.
(2)

The sum of the first n terms of the series is 2750.

(c) Show that n is given by

$$n^2 - 15n = 55 \times 40.$$

(4)

(d) Hence find the value of n.
(3)

a) $a + 17d = 25$ (1)
$a + 20d = 32.5$ (2)

b) Solve (1) and (2) simultaneously

(2) − (1) gives
$3d = 7.5$
$d = 2.5$

put $d = 2.5$ in (2)
$a + 20(2.5) = 32.5$
$a + 50 = 32.5$
$a = 32.5 - 50$
$a = -17.5$

c) $S_n = \frac{n}{2}(2a + (n-1)d)$

$2750 = \frac{n}{2}(2 \times -17.5 + (n-1)2.5)$
$2750 = \frac{n}{2}\left(-35 + \frac{5}{2}n - \frac{5}{2}\right)$
(x through by 4)
$4 \times 2750 = \cancel{4}^{2} \times \frac{n}{\cancel{2}_{1}}\left(-35 + \frac{5}{2}n - \frac{5}{2}\right)$

9c continued

$11000 = -70n + 5n^2 - 5n$

$0 = 5n^2 - 75n - 11000$

$0 = 5(n^2 - 15n - 2200)$

$\therefore n^2 - 15n - 2200 = 0$

but $2200 = 55 \times 40$

$\therefore n^2 - 15n = 55 \times 40$

as required

$$\begin{array}{r} 55 \\ {}_{2}40\times \\ \hline 2200 \\ \hline \end{array}$$

$$\begin{array}{r} 2750 \\ {}^{3}\ {}^{2}\ \ 4\times \\ \hline 11000 \\ \hline \end{array}$$

$$5 \overline{)11000}\quad 2200$$

d) $n^2 - 15n - 2200 = 0$

$(n - 55)(n + 40) = 0$

Either $n = 55$ or $n = -40$

$\therefore n = 55$ (cannot be negative)

Leave blank

4. A sequence a_1, a_2, a_3,... is defined by

$$a_1 = 2$$
$$a_{n+1} = 3a_n - c$$

where c is a constant.

(a) Find an expression for a_2 in terms of c.

(1)

Given that $\sum_{i=1}^{3} a_i = 0$

(b) find the value of c.

(4)

a) $a_2 = 3a_1 - c$
$a_2 = 3 \times 2 - c$
$a_2 = 6 - c$

b) $\sum_{i=1}^{3} a_i = 0$ means

$a_1 + a_2 + a_3 = 0$ (1)

$a_3 = 3a_2 - c$
$a_3 = 3(6-c) - c$
$a_3 = 18 - 3c - c$
$= 18 - 4c$

in (1) gives
$2 + (6-c) + (18-4c] = 0$
$2 + 6 - c + 18 - 4c = 0$
$26 = 5c$
$c = \frac{26}{5} = 5\frac{1}{5}$

C1 JAN 2011

Leave blank

6. An arithmetic sequence has first term a and common difference d. The sum of the first 10 terms of the sequence is 162.

(a) Show that $10a + 45d = 162$

(2)

Given also that the sixth term of the sequence is 17,

(b) write down a second equation in a and d,

(1)

(c) find the value of a and the value of d.

(4)

a) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = 162 = \frac{10}{2}(2a + (10-1)d)$

$162 = 5(2a + 10d - d)$

$\therefore\ 162 = 10a + 45d$ (1) as required

b) Sixth term is $a + 5d$

$a + 5d = 17$ (2)

c) (2) gives $a = 17 - 5d$

sub in (1)

$162 = 10(17 - 5d) + 45d$

$162 = 170 - 50d + 45d$

$5d = 8$

$d = \frac{8}{5}$

in (2) gives

$a = 17 - \left(5 \times \frac{8}{5}\right) = 17 - 8 = 9$

$\therefore\ a = 9,\ d = \frac{8}{5}$

Leave blank

4. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1$$

$$x_{n+1} = ax_n + 5, \qquad n \geqslant 1$$

where a is a constant.

(a) Write down an expression for x_2 in terms of a.

(1)

(b) Show that $x_3 = a^2 + 5a + 5$

(2)

Given that $x_3 = 41$

(c) find the possible values of a.

(3)

a) $x_1 = 1$

$x_2 = ax_1 + 5 = a + 5$

b) $x_3 = ax_2 + 5$

$= a(a+5) + 5$

$= a^2 + 5a + 5$ (as required)

c) $41 = a^2 + 5a + 5$

$0 = a^2 + 5a - 36$

$0 = (a+9)(a-4)$

$a = -9, \; a = 4$

C1 Jan 2012

Leave blank

9. A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is £P.
Salary increases by £$(2T)$ each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is £$(P + 1800)$.
Salary increases by £T each year, forming an arithmetic sequence.

(a) Show that the **total** earned under Salary Scheme 1 for the 10-year period is

$$£(10P + 90T)$$

(2)

For the 10-year period, the **total** earned is the same for both salary schemes.

(b) Find the value of T.

(4)

For this value of T, the salary in Year 10 under Salary Scheme 2 is £29 850

(c) Find the value of P.

(3)

a) $a = P, \quad d = 2T$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = 5(2P + 9 \times 2T)$

$= 10P + 90T$ ① (as required)

b) For scheme 2

$a = P + 1800, \quad d = T$

$36 \times 5 = 180$

$S_{10} = 5(2P + 3600 + 9T)$

$= 10P + 18000 + 45T$ ②

If the same then ① = ②

$\cancel{10P} + 90T = \cancel{10P} + 18000 + 45T$

$45T = 18000$

$T = 400$

$4 \times 45 = 180$

$400 \times 45 = 18000$

c) $a + 9d = 29850$

$P + 1800 + 9T = 29850$

$P + 1800 + 9 \times 400 = 29850$

$P + 1800 + 3600 = 29850$

$P + 5400 = 29850$

$P = 24450$

$29850 - 5400 = 24450$

Leave blank

4. A girl saves money over a period of 200 weeks. She saves 5p in Week 1, 7p in Week 2, 9p in Week 3, and so on until Week 200. Her weekly savings form an arithmetic sequence.

(a) Find the amount she saves in Week 200.

(3)

(b) Calculate her total savings over the complete 200 week period.

(3)

a) $5, 7, 9, \ldots$

$n = 200$

$d = 2$

$a = 5$

Arithmetic progression

nth term $= a + (n-1)d$

Amount in Week 200

$= 5 + (200-1)2$

$= 403$ pence

$= £4.03$

Learn these formulae

b) Total savings over 200 weeks $= S_{200}$

$S_{200} = \frac{200}{2}(5 + 403)$

$S_n = \frac{n}{2}(a + l)$

$= 100 \times 408$

$= 40800$ pence

$= £408$

Alternative formula

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{200} = \frac{200}{2}[10 + 199 \times 2]$

$= 100(408)$

$= 40800$ pence

$= £408$

Leave blank

8. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 3a_n + 5, \qquad n \geqslant 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 9k + 20$.

(2)

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 10.

(4)

a) $a_1 = k$

$a_{n+1} = 3a_n + 5$ | with $n = 1$

$a_2 = 3a_1 + 5$

$\therefore a_2 = 3k + 5$

b) $a_3 = 3a_2 + 5$ with $n = 2$

$a_3 = 3(3k+5) + 5$

$a_3 = 9k + 15 + 5$

$a_3 = 9k + 20$

c) $\sum_{r=1}^{4} a_r = a_1 + a_2 + a_3 + a_4$

a_4

$\therefore = k + (3k+5) + (9k+20) + 3(9k+20) + 5$

$= k + 3k + 5 + 9k + 20 + 27k + 65$

$= 40k + 90$

d) $\sum_{r=1}^{4} a_r = 40k + 90$

$= 10(4k + 9)$

Since 10 is a factor then $\sum_{r=1}^{4} a_r$ is divisible by 10

Leave blank

5. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, \quad n \geqslant 1,$$

where a is a constant.

(a) Find an expression for x_2 in terms of a.

(1)

(b) Show that $x_3 = a^2 - 3a - 3$.

(2)

Given that $x_3 = 7$,

(c) find the possible values of a.

(3)

$x_{n+1} = a\ x_n - 3$ $\qquad$ $x_1 = 1$ given

a) when $n = 1$

$x_2 = ax_1 - 3$

$x_2 = a \times 1 - 3$

$x_2 = a - 3$

b) show $x_3 = a^2 - 3a - 3$

let $n = 2$

$x_{2+1} = ax_2 - 3$

$x_3 = ax_2 - 3$

$x_3 = a(a-3) - 3$

$x_3 = a^2 - 3a - 3$

c) Given $x_3 = 7$

$x_3 = a^2 - 3a - 3$

$7 = a^2 - 3a - 3$

$0 = a^2 - 3a - 10$

$0 = (a-5)(a+2)$

Either $a - 5 = 0$ or $a + 2 = 0$

$a = 5$ or $a = -2$

Leave blank

7. Sue is training for a marathon. Her training includes a run every Saturday starting with a run of 5 km on the first Saturday. Each Saturday she increases the length of her run from the previous Saturday by 2 km.

(a) Show that on the 4th Saturday of training she runs 11 km.

(1)

(b) Find an expression, in terms of n, for the length of her training run on the nth Saturday.

(2)

(c) Show that the total distance she runs on Saturdays in n weeks of training is $n(n + 4)$ km.

(3)

On the nth Saturday Sue runs 43 km.

(d) Find the value of n.

(2)

(e) Find the total distance, in km, Sue runs on Saturdays in n weeks of training.

(2)

a) 5, 7, 9, 11 km — 2km more each Saturday

b) distance on n^{th} Saturday
$= 5 + (n-1)2$
$= 5 + 2n - 2$
$= 3 + 2n$

$a, a+d, a+2d$ are terms in arithmetic progression

n^{th} term $= a + (n-1)d$ where $a = 5$, $d = 2$

c) Total distance in n weeks

$= \frac{n}{2}\left[10 + (n-1)2\right]$

$= \frac{n}{2}(10 + 2n - 2)$

$= \frac{n}{2}(8 + 2n)$

$= 4n + n^2$

$= n(n+4)$

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

7d) distance on n^{th} Saturday

$= 3+2n$ (from part (b))

When distance $= 43$

$\therefore 43 = 3+2n$

$\therefore 40 = 2n$

$\therefore n = 20$

e) $n = 20$

From part (c),

Total distance $= 20(20+4)$

$= 20 \times 24$

$= 480$ km

Leave blank

5. A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

↓ Year 10

(a) the value of d, **(3)**

(b) the value of a, **(2)**

(c) the total number of houses built in Oldtown over the 40-year period. **(3)**

	1st	10th	40th
a)	1951	1960	1990
	a	$a+9d$	$a+39d$

$a + 9d = 2400$ ①

$a + 39d = 600$ ②

② − ① $30d = -1800$

$d = -60$

put $d = -60$ in ①

$a + 9\times(-60) = 2400$

$a = 2400 + 540$

b) $a = 2940$

c) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{40} = \frac{40}{2}(2\times2940 + (39\times-60))$

$= 20(5880 - 2340)$

$= 20 \times 3540$

$= 70800$

39
6 ×
234

2940
2 ×
5880

5880
2340 −
3540

3540
2 ×
7080

Leave blank

7. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$

$$a_{n+1} = 2a_n - 7, \qquad n \geqslant 1,$$

where k is a constant.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 4k - 21$.

(2)

Given that $\sum_{r=1}^{4} a_r = 43$,

(c) find the value of k.

(4)

a) $a_1 = k$

$a_2 = 2a_1 - 7$

$a_2 = 2k - 7$

b) $a_3 = 2a_2 - 7$

$a_3 = 2(2k-7) - 7$

$a_3 = 4k - 14 - 7$

$a_3 = 4k - 21$

c) $a_4 = 2a_3 - 7$

$a_4 = 2(4k-21) - 7$

$a_4 = 8k - 42 - 7$

$a_4 = 8k - 49$

$\sum_{r=1}^{4} a_r = 43 = a_1 + a_2 + a_3 + a_4$

$43 = k + (2k-7) + (4k-21) + (8k-49)$

$43 = 15k - 77$

$43 + 77 = 15k$

$120 = 15k$

$k = 8$

$15 \overline{)120}$ 8

May 2006

Leave blank

4. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3,$$
$$a_{n+1} = 3a_n - 5, \quad n \geqslant 1.$$

(a) Find the value of a_2 and the value of a_3.

(2)

(b) Calculate the value of $\sum_{r=1}^{5} a_r$.

(3)

a)

$a_2 = 3a_1 - 5 = 3\times 3 - 5 = 9 - 5 = 4$

$a_3 = 3a_2 - 5 = 3\times 4 - 5 = 12 - 5 = 7$

b) $\sum_{r=1}^{5} a_r = 3 + 4 + 7 + a_4 + a_5$

$a_4 = 3a_3 - 5 = 3\times 7 - 5 = 21 - 5 = 16$

$a_5 = 3a_4 - 5 = 3\times 16 - 5 = 48 - 5 = 43$

$\sum_{r=1}^{5} a_r = 3 + 4 + 7 + 16 + 43 = \underline{\underline{73}}$

May 2010

Leave blank

5. A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{(a_n^2 + 3)}, \qquad n \geqslant 1,$$
$$a_1 = 2$$

(a) Find a_2 and a_3, leaving your answers in surd form.

(2)

(b) Show that $a_5 = 4$

(2)

a) $a_2 = \sqrt{a_1^2 + 3}$

$a_2 = \sqrt{2^2 + 3}$

$a_2 = \sqrt{7}$

$a_3 = \sqrt{a_2^2 + 3}$

$a_3 = \sqrt{(\sqrt{7})^2 + 3}$

$a_3 = \sqrt{7 + 3} = \sqrt{10}$

b) $a_4 = \sqrt{(a_3)^2 + 3}$

$a_4 = \sqrt{(\sqrt{10})^2 + 3}$

$a_4 = \sqrt{10 + 3}$

$a_4 = \sqrt{13}$

$a_5 = \sqrt{(a_4)^2 + 3}$

$a_5 = \sqrt{(\sqrt{13})^2 + 3}$

$a_5 = \sqrt{13 + 3}$

$a_5 = \sqrt{16}$

$a_5 = 4$ as required

H 3 5 3 8 3 A 0 8 2 8

May 2010

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9. A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season. He pays £a for their first day, £$(a+d)$ for their second day, £$(a+2d)$ for their third day, and so on, thus increasing the daily payment by £d for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.

(a) Use this information to form an equation in a and d.

(2)

A picker who works for all 30 days will earn a total of £1005

(b) Show that $15(a+40.75)=1005$

(2)

(c) Hence find the value of a and the value of d.

(4)

a) $a+29d=40.75$ (1)

b) $S_{30}=\frac{30}{2}\left(2a+(30-1)d\right)=1005$ (2)

From part a) $a+29d=40.75$

$d=\frac{40.75-a}{29}$

Put this in (2)

$\frac{\cancel{30}^{15}}{\cancel{2}_1}\left(2a+\cancel{29}^1\left(\frac{40.75-a}{\cancel{29}_1}\right)\right)=1005$

$15\left(2a+(40.75-a)\right)=1005$

$15(a+40.75)=1005$

as required.

c) $a+40.75=\frac{1005}{15}$ $\quad 60\times15=900$, $7\times15=105$

$a+40.75=67$

$a=67-40.75$

$a=26.25$

(1) gives $29d=40.75-26.25$

$29d=14.5$

$d=\frac{14.5}{29}=\frac{1}{2}=0.5$

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$\therefore a=£26.25,\ d=£0.50$

May 2011

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5. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$
$$a_{n+1} = 5a_n + 3, \qquad n \geqslant 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 25k + 18$.

(2)

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k, in its simplest form.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 6.

(4)

a) $a_1 = k$

$a_2 = 5a_1 + 3$

$a_2 = 5k + 3$

b) $a_3 = 5a_2 + 3$
$a_3 = 5(5k+3) + 3$
$a_3 = 25k + 15 + 3$
$a_3 = 25k + 18$ as required

c) $\sum_{r=1}^{4} a_r = a_1 + a_2 + a_3 + a_4$

$a_4 = 5a_3 + 3$
$a_4 = 5(25k+18) + 3$
$= 125k + 90 + 3$
$= 125k + 93$

$\sum_{r=1}^{4} a_r = k + (5k+3) + (25k+18) + 125k + 93$
$= 156k + 114$

d) $156k + 114 = 6(26k + 19)$ which is a factor of 6

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9. (a) Calculate the sum of all the even numbers from 2 to 100 inclusive, 50 terms

$$2 + 4 + 6 + \ldots\ldots + 100$$

(3)

(b) In the arithmetic series

$$k + 2k + 3k + \ldots\ldots + 100$$

k is a positive integer and k is a factor of 100.

(i) Find, in terms of k, an expression for the number of terms in this series.

(ii) Show that the sum of this series is

$$50 + \frac{5000}{k}$$

(4)

(c) Find, in terms of k, the 50th term of the arithmetic sequence

$$(2k + 1), (4k + 4), (6k + 7), \ldots\ldots,$$

giving your answer in its simplest form.

(2)

a) $a = 2$
$d = 2$

$S_{50} = \frac{50}{2}(2\times 2 + (49)2)$

$= \frac{50}{2}(4 + 98)$

$= 100 + (50 \times 49)$

$= 100 + 2450$

$= 2550$

$49 \times 5 = 245$

b) $a = k$
$d = k$

$S_n = \frac{n}{2}(2k + (n-1)k)$

9b) continued May 2011

(i) 1st 2nd 3rd n^{th}

k $2k$ $3k$ 100

a $a+d$ $a+2d$ $a+(n-1)d$

$a=k$
$d=k$

$$100 = k + (n-1)k$$

$$100 = k + nk - k$$

$$n = \frac{100}{k}$$

(ii) $S_n = \frac{n}{2}(2k + (n-1)k)$

Sub $n = \frac{100}{k}$

$$S_n = \frac{100}{2k}\left(2k + \left(\frac{100}{k} - 1\right)k\right)$$

$$S_n = \frac{100}{2k}(2k + 100 - k)$$

$$= \frac{100}{2k}(k + 100)$$

$$= 50 + \frac{5000}{k} \quad \text{a/ req}$$

c) a $a+d$ $a+2d$

$2k+1$ $4k+4$ $6k+7$

$d = (4k+4) - (2k+1) = 2k+3$

50^{th} term is $a + 49d$

$$= (2k+1) + 49(2k+3)$$

$$= 2k + 98k + 1 + 147$$

$$= 100k + 148$$

Leave blank

5. A sequence of numbers $a_1, a_2, a_3 \ldots$ is defined by

$$a_1 = 3$$

$$a_{n+1} = 2a_n - c \qquad (n \geqslant 1)$$

where c is a constant.

(a) Write down an expression, in terms of c, for a_2

(1)

(b) Show that $a_3 = 12 - 3c$

(2)

Given that $\sum_{i=1}^{4} a_i \geqslant 23$

(c) find the range of values of c.

(4)

a) $a_1 = 3$
$a_2 = 2a_1 - c$
$a_2 = 2 \times 3 - c$
$a_2 = 6 - c$

b) $a_3 = 2a_2 - c$
$a_3 = 2(6-c) - c$
$a_3 = 12 - 2c - c$
$a_3 = 12 - 3c$ (as required)

c) $\sum_{i=1}^{4} a_i \geqslant 23$

$a_1 + a_2 + a_3 + a_4 \geqslant 23$

$a_4 = 2a_3 - c$
$a_4 = 2(12-3c) - c$
$a_4 = 24 - 6c - c$
$a_4 = 24 - 7c$

So
$3 + (6-c) + (12-3c) + (24-7c) \geqslant 23$
$3 + 6 - c + 12 - 3c + 24 - 7c \geqslant 23$
$45 - 11c \geqslant 23$
$45 - 23 \geqslant 11c$

Leave blank

Question 5 continued

5c (continued)

$22 \geq 11c$

$11c \leq 22$

$c \leq 2$

Q5

(Total 7 marks)

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6. A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(a) Find how much he saves in week 15 **(2)**

(b) Calculate the total amount he saves over the 60 week period. **(3)**

The boy's sister also saves some money each week over a period of m weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence. She saves a total of £63 in the m weeks.

(c) Show that

$$m(m + 1) = 35 \times 36$$

(4)

(d) Hence write down the value of m. **(1)**

a) 1st 2nd 3rd $a = 10$
10 15 20 $d = 5$
a $a+d$ $a+2d$

15^{th} week $a + 14d$
$= 10 + 14 \times 5 = 80p$

b) $S_n = \frac{n}{2}(2a + (n-1)d)$
$S_{60} = \frac{60}{2}(2 \times 10 + 59 \times 5)$

$59 \times 5 = 295$

$= 30(20 + 295)$
$= 30 \times 315$
$= 9450p$
$= £94.50$

$315 \times 3 = 945$

P 4 0 6 8 4 A 0 1 0 2 4

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Question 6 continued

c) 1st 2nd 3rd $a=10$

10 20 30 $d=10$

a $a+d$ $a+2d$

$S_m = \frac{m}{2}(2a+(m-1)d)$

$6300 = \frac{m}{2}(20+10(m-1))$

$12600 = m(20+10m-10)$

$12600 = m(10m+10)$

$0 = 10m^2 + 10m - 12600$

$0 = m^2 + m - 1260$

$$\begin{array}{r} 35\times \\ 36 \\ \hline 210 \\ 1050 \\ \hline 1260 \end{array}$$

$m^2 + m = 1260$

$m(m+1) = 35 \times 36$ (as required)

d) $0 = (m-35)(m+36)$

$m = 35$ or $m = -36$

as m must be positive

$m = 35$

Turn over

Leave blank

4. A sequence $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+1} = 2u_n - 1, \quad n \geqslant 1$$

Given that $u_2 = 9$,

(a) find the value of u_3 and the value of u_4,

(2)

(b) evaluate $\sum_{r=1}^{4} u_r$.

(3)

a) $u_2 = 9$

$u_3 = 2u_2 - 1$

$u_3 = 2 \times 9 - 1 = 17$

$u_4 = 2u_3 - 1$

$u_4 = 2 \times 17 - 1 = 33$

b) $\sum_{r=1}^{4} u_r = u_1 + u_2 + u_3 + u_4$

$u_{n+1} = 2u_n - 1$

$u_2 = 2u_1 - 1$

$9 = 2u_1 - 1$

$9 + 1 = 2u_1$

$10 = 2u_1$

$u_1 = 5$

$\sum_{r=1}^{4} u_r = 5 + 9 + 17 + 33$

$= 64$

Leave blank

7. Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(a) Find the number of points that Lewis scored for capturing his 20th spaceship.
(2)

(b) Find the total number of points Lewis scored for capturing his first 20 spaceships.
(3)

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her nth dragon,

(c) find the value of n.
(3)

1st	2nd	3rd
140	160	180

a) $a = 140, \ d = 20$

$a_{20} = a + 19d$

$= 140 + 19 \times 20$

$= 140 + 380$

$= \underline{\underline{520}}$

b) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = \frac{20}{2}(2 \times 140 + 19 \times 20)$

$S_{20} = 10(280 + 380)$

$= 10 \times 660$

$= \underline{\underline{6600}}$

C1 Jan 2013

7c) $a = 300$, $L = 700$, $S_n = 300$

$S_n = \frac{n}{2}(a + L)$ ← alternative version of formula

$8500 = \frac{n}{2}(300 + 700)$

$2 \times 8500 = 1000n$

$17000 = 1000n$

$\underline{\underline{n = 17}}$

C1 MAY 2013

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4. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4$$
$$a_{n+1} = k(a_n + 2), \quad \text{for } n \geqslant 1$$

where k is a constant.

(a) Find an expression for a_2 in terms of k.

(1)

Given that $\sum_{i=1}^{3} a_i = 2$,

(b) find the two possible values of k.

(6)

a) $a_2 = k(a_1 + 2)$
$= k(4 + 2)$
$= \underline{\underline{6k}}$

b) $a_3 = k(a_2 + 2)$
$= k(6k + 2)$
$= \underline{\underline{6k^2 + 2k}}$

$\sum_{i=1}^{3} a_i = a_1 + a_2 + a_3 = 2$

So $4 + 6k + (6k^2 + 2k) = 2$
$6k^2 + 8k + 4 = 2$
$6k^2 + 8k + 2 = 0$
$2(3k^2 + 4k + 1) = 0$
$2(3k + 1)(k + 1) = 0$

Either $3k + 1 = 0$ or $k + 1 = 0$
$\underline{\underline{k = -\frac{1}{3}}}$ $\underline{\underline{k = -1}}$

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C1 MAY 2013

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7. A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week N.

(a) Find the value of N.

(2)

The company then plans to continue to make 600 mobile phones each week.

(b) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.

(5)

Week	1	2	3	N
phones	200	220	240	600
	a	$a+d$	$a+2d$	$a+(N-1)d$

$a = 200, \quad d = 20$

a) in week N

$$a + (N-1)d = 600$$
$$200 + (N-1)20 = 600$$
$$200 + 20N - 20 = 600$$
$$20N = 600 - 200 + 20$$
$$20N = 420$$
$$\underline{\underline{N = 21}}$$

b) First 21 weeks sum

$$S_n = \frac{n}{2}(2a + (n-1)d) \text{ or } S_n = \frac{n}{2}(a+l)$$

$$S_{21} = \frac{21}{2}(200 + 600) = 21 \times 400$$
$$= 8400$$

From week 22 to 52 there are 31 weeks

$31 \times 600 = 18600$

Total

$$\begin{array}{r} 8400 \\ 18600 \\ \hline \underline{\underline{27000}} \end{array}$$

Total for 52 weeks is 27000 phones

P 4 1 8 0 2 A 0 1 2 2 8