

2. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$.

(2)

a) $y = e^{2x} \tan x$

Product rule $u = e^{2x}$ $v = \tan x$

$$\frac{du}{dx} = 2e^{2x} \quad \frac{dv}{dx} = \sec^2 x$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$= e^{2x} \sec^2 x + 2e^{2x} \tan x$$

Turning point $\frac{dy}{dx} = 0$

$$0 = \sec^2 x + 2 \tan x \quad [e^{2x} \neq 0]$$

$$0 = \tan^2 x + 1 + 2 \tan x \quad [\sec^2 x = \tan^2 x + 1]$$

$$0 = \tan^2 x + 2 \tan x + 1$$

$$0 = (\tan x + 1)^2$$

$$\Rightarrow \tan x = -1.$$

b) When $x = 0$ $y = 0$, $x = 0$ $\frac{dy}{dx} = 1 \Rightarrow y = x$

1. (a) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2 \sqrt{5x - 1}.$$

(6)

- (b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x .

(4)

a) $y = x^2 (5x - 1)^{1/2}$

Product Rule $u = x^2$ $v = (5x - 1)^{1/2}$

$$\frac{du}{dx} = 2x$$

$$\frac{dv}{dx} = \frac{1}{2} (5x - 1)^{-1/2} \times 5$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$= \frac{5}{2} x^2 (5x - 1)^{-1/2} + (5x - 1)^{1/2} \times 2x$$

(no need to simplify)

At $x = 2$ $\frac{dy}{dx} = \frac{5}{2} (2^2) (10 - 1)^{-1/2} + (10 - 1)^{1/2} \times 4$

$$\frac{dy}{dx} = \frac{46}{3}$$

b) Quotient Rule.

$$u = \sin 2x \quad v = x^2$$

$$\frac{du}{dx} = 2 \cos 2x \quad \frac{dv}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \frac{2x^2 \cos 2x - 2x \sin 2x}{x^4}$$



4. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

$$\frac{dx}{dy} = -2 \sin(2y + \pi)$$

$$\frac{dy}{dx} = -\frac{1}{2 \sin(2y + \pi)}$$

At $\frac{\pi}{4}$

$$\frac{dy}{dx} = -\frac{1}{2 \sin(\frac{3\pi}{2})}$$

$$= \frac{1}{2}$$

$$\text{Equation } (y - y_1) = m(x - x_1)$$

$$\text{At } (0, \frac{\pi}{4}) \quad y - \frac{\pi}{4} = \frac{1}{2}x$$

$$y = \frac{1}{2}x + \frac{\pi}{4}$$

$$a = \frac{1}{2}$$

$$b = \frac{\pi}{4}$$



7.

$$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P .

- (a) Find the exact coordinates of P .

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

- (b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1 , x_2 and x_3 .

(3)

- (c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

a) Turning point when $f'(x) = 0$

Differentiate with product rule

$$\frac{dy}{dx} = 3e^x + 3xe^x$$

$$3e^x + 3xe^x = 0$$

$$3e^x(1+x) = 0 \quad e^x \text{ cannot } = 0$$

$$x = -1 \quad \text{substitute into } f \text{ but for } y \text{ value}$$

$$f(-1) = -3e^{-1} - 1 \quad (-1, -3e^{-1} - 1)$$

b) $x_1 = 0.2596 \quad x_2 = 0.2571 \quad x_3 = 0.2578$

(use ANS button to avoid rounding error)

c) Substitute in 0.25755 and 0.25765 to show a change of sign.

$$f(0.25755) = -0.00038 \quad f(0.25765) = 0.000109 \quad \text{Change of sign}$$

$\Rightarrow x = 0.2576$
is a root.



4. (i) Given that $y = \frac{\ln(x^2+1)}{x}$, find $\frac{dy}{dx}$. (4)

(ii) Given that $x = \tan y$, show that $\frac{dy}{dx} = \frac{1}{1+x^2}$. (5)

$$y = \frac{\ln(x^2+1)}{x} \quad \text{Quotient Rule.}$$

$$u = \ln(x^2+1) \quad v = x$$

$$\frac{du}{dx} = \frac{2x}{x^2+1} \quad \frac{dv}{dx} = 1$$

$$\frac{dy}{dx} = v \frac{du}{dx} - u \frac{dv}{dx}$$

$$v^2$$

$$= x \left(\frac{2x}{x^2+1} \right) - \ln(x^2+1) \cdot 1$$

$$x^2$$

$$= \frac{2}{x^2+1} - \frac{1}{x^2} \ln(x^2+1)$$

(ii) $x = \tan y$

$$\frac{dx}{dy} = \sec^2 y$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} \quad (\sec^2 y = 1 + \tan^2 y)$$

$$\frac{dy}{dx} = \frac{1}{1+\tan^2 y}$$

$$(x = \tan y)$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

7. (a) By writing $\sec x$ as $\frac{1}{\cos x}$, show that $\frac{d(\sec x)}{dx} = \sec x \tan x$.

(3)

Given that $y = e^{2x} \sec 3x$,

(b) find $\frac{dy}{dx}$.

(4)

The curve with equation $y = e^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (a, b) .

- (c) Find the values of the constants a and b , giving your answers to 3 significant figures.

(4)

(a) $y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$

$$y = (\cos x)^{-1}$$

$$\frac{dy}{dx} = -(\cos x)^{-2} \cdot (-\sin x)$$

$$= \frac{\sin x}{\cos^2 x} \quad \left(= \frac{\sin x}{\cos x \cos x}\right)$$

$$\frac{dy}{dx} = \sec x \tan x$$

(b) $y = e^{2x} \sec 3x$

Product Rule $u = e^{2x}$ $v = \sec 3x$

$$\frac{du}{dx} = 2e^{2x} \quad \frac{dv}{dx} = 3\sec 3x \tan 3x$$

$$\frac{dy}{dx} = 2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan x$$

Question 7 continued

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c) Turning point $\frac{dy}{dx} = 0$

$$e^{2x} \sec 3x (2 + 3 \tan 3x) = 0$$

$$(e^{2x} \neq 0 \quad \sec 3x \neq 0)$$

$$2 + 3 \tan 3x = 0$$

$$\tan 3x = -\frac{2}{3}$$

$$3x = -0.588$$

$$x = -0.196$$

Substitute to find y

$$y = e^{2(-0.196)} \sec(3x - 0.196)$$
$$= 0.812$$

$$x = -0.196 \quad y = 0.812$$

7. The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}.$$

(a) Show that

$$\frac{dy}{dx} = \frac{6 \sin 2x + 4 \cos 2x + 2}{(2 + \cos 2x)^2} \quad (4)$$

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

a) quotient rule.

$$u = 3 + \sin 2x \quad v = 2 + \cos 2x$$

$$\frac{du}{dx} = 2 \cos 2x \quad \frac{dv}{dx} = -2 \sin 2x$$

$$\frac{dy}{dx} = \frac{(2 + \cos 2x)(2 \cos 2x) - (-2 \sin 2x)(3 + \sin 2x)}{(2 + \cos 2x)^2}$$

$$= \frac{4 \cos^2 2x + 2 \cos^2 2x + 6 \sin 2x + 2 \sin^2 2x}{(2 + \cos 2x)^2}$$

$$= \frac{4 \cos^2 2x + 6 \sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$$

$$= \frac{4 \cos^2 2x + 6 \sin 2x + 2}{(2 + \cos 2x)^2}$$

$$(\cos^2 2x + \sin^2 2x = 1)$$

(b) When $x = \pi/2$ $y = \frac{3 + \sin 2(\pi/2)}{2 + \cos 2(\pi/2)}$

Substitute x

\downarrow $y = 3$

$$x = \pi/2 \quad \frac{dy}{dx} = -2$$

$$\Rightarrow y - 3 = -2(x - \pi/2)$$

$$y - 3 = -2x + \pi$$

$$y + 2x = \pi + 3$$

$$y = -2x + \pi + 3$$

$$a = -2$$

$$b = \pi + 3$$

8. Given that

$$\frac{d}{dx}(\cos x) = -\sin x,$$

(a) show that $\frac{d}{dx}(\sec x) = \sec x \tan x.$

(3)

Given that

$$x = \sec 2y,$$

(b) find $\frac{dx}{dy}$ in terms of $y.$

(2)

(c) Hence find $\frac{dy}{dx}$ in terms of $x.$

(4)

TOTAL FOR PAPER: 75 MARKS

END

8)

$$\begin{aligned}\frac{d}{dx}(\sec x) &= \frac{d}{dx}\left(\frac{1}{\cos x}\right) \\ &= \frac{d}{dx}(\cos x)^{-1} \\ &= -(\cos x)^{-2} \cdot -\sin x\end{aligned}$$

$$\begin{aligned}&= \frac{\sin x}{(\cos x)^2} \\ &= \frac{\sin x}{(\cos x)(\cos x)} = \frac{\tan x \sec x}{\cos^2 x} \quad \rightarrow\end{aligned}$$

(b) $x = \sec 2y$

$$\frac{dx}{dy} = 2\tan 2y \sec 2y$$

$$(c) \frac{dy}{dx} = \frac{1}{2\tan^2 y \sec^2 y}$$

but

$$x = \sec^2 y$$

$$\frac{dy}{dx} = \frac{1}{2x \tan^2 y}$$

but

$$1 + \tan^2 y = \sec^2 y$$

$$\text{se } 1 + \tan^2 2y = \sec^2 2y$$

$$\tan^2 2y = \sec^2 2y - 1$$

$x = \sec^2 y$

$$\tan^2 2y = x^2 - 1$$

$$\tan^2 2y = \sqrt{x^2 - 1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2x \sqrt{x^2 - 1}}$$

1. Differentiate with respect to x , giving your answer in its simplest form,

(a) $x^2 \ln(3x)$ (4)

(b) $\frac{\sin 4x}{x^3}$ (5)

(a) Product Rule

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$u = x^2 \quad v = \ln(3x)$$

$$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = 2x \ln(3x) + x^2 \cdot \frac{1}{x}$$

$$= 2x \ln(3x) + x$$

(b) Quotient Rule.

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$u = \sin 4x \quad v = x^3$$

$$\frac{du}{dx} = 4 \cos 4x \quad \frac{dv}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{4x^3 \cos 4x - 3x^2 \sin 4x}{x^6}$$

$$= \frac{4x \cos 4x - 3 \sin 4x}{x^4}$$



4. The point P is the point on the curve $x = 2 \tan\left(y + \frac{\pi}{12}\right)$ with y -coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P .

(7)

$$4) \quad \frac{dx}{dy} = 2 \sec^2\left(y + \frac{\pi}{12}\right)$$

$$\text{when } y = \frac{\pi}{4} \quad \frac{dx}{dy} = \frac{2}{\cos^2\left(\frac{\pi}{4} + \frac{\pi}{12}\right)}$$

$$= 8$$

$$\frac{dy}{dx}_{\text{TANGENT}} = \frac{1}{8}$$

$$\frac{dy}{dx}_{\text{normal}} = -8$$

$$x = 2 \tan\left(\frac{\pi}{4} + \frac{\pi}{12}\right)$$

$$= 2\sqrt{3}$$

$$y = \frac{\pi}{4} \quad (\text{given})$$

$$y - \frac{\pi}{4} = -8(x - 2\sqrt{3})$$



3. A curve C has equation

$$y = x^2 e^x.$$

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation.

(3)

(b) Hence find the coordinates of the turning points of C .

(3)

(c) Find $\frac{d^2y}{dx^2}$.

(2)

(d) Determine the nature of each turning point of the curve C .

(2)

a) $u = x^2 \quad v = e^x$

$$\frac{dy}{dx} = x^2 e^x + 2x e^x$$

$$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = e^x \quad \frac{dy}{dx}$$

b) $\frac{dy}{dx} = 0 \quad 0 = (x^2 + 2x) e^x$

$$x^2 + 2x = 0 \quad e^x \neq 0$$

$$x = 0 \quad x = -2$$

sub. x values into

$$y = 0 \quad xy = 4e^{-2}$$

$$y = x^2 e^x$$

$$(0, 0) \quad (-2, 4e^{-2})$$

c) $\frac{d^2y}{dx^2} = x^2 e^x + 2x e^x + 2x e^x + 2 e^x$
 $= e^x (x^2 + 4x + 2)$

product rule

d) $x = 0 \quad \frac{d^2y}{dx^2} > 0 \quad \underline{\text{minimum}}$

$$x = -2 \quad \frac{d^2y}{dx^2} < 0 \quad \underline{\text{maximum}}$$



6. (a) Differentiate with respect to x ,

(i) $e^{3x}(\sin x + 2 \cos x)$, (3)

(ii) $x^3 \ln(5x + 2)$. (3)

Given that $y = \frac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(b) show that $\frac{dy}{dx} = \frac{20}{(x+1)^3}$. (5)

(c) Hence find $\frac{d^2y}{dx^2}$ and the real values of x for which $\frac{d^2y}{dx^2} = -\frac{15}{4}$. (3)

6(a) Product Rule

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$u = e^{3x} \quad v = \sin x + 2 \cos x$$

$$\frac{du}{dx} = 3e^{3x} \quad \frac{dv}{dx} = \cos x - 2 \sin x$$

$$\frac{dy}{dx} = e^{3x} (\cos x - 2 \sin x) + (\sin x + 2 \cos x) 3e^{3x}$$

(ii) Product Rule

$$u = x^3 \quad v = \ln(5x+2)$$

$$\frac{du}{dx} = 3x^2 \quad \frac{dv}{dx} = \frac{5}{5x+2}$$

$$\frac{dy}{dx} = \frac{5x^3}{5x+2} + 3x^2 (\ln(5x+2))$$

b) Quotient Rule

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$u = 3x^2 + 6x - 7 \quad v = (x+1)^2$$

$$\frac{du}{dx} = 6x + 6 \quad \frac{dv}{dx} = 2(x+1)$$

$$\frac{dy}{dx} = \frac{(6x+6)(x+1)^2 - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$$

cancel $(x+1)$ everywhere

$$= \frac{(6x+6)(x+1) - 2(3x^2+6x-7)}{(x+1)^3}$$
$$= \frac{6x^2 + 6x + 6x + 6 - 6x^2 - 12x + 14}{(x+1)^3}$$
$$= \frac{20}{(x+1)^3}$$

(c)

$$\frac{dy}{dx} = 20(x+1)^{-3}$$

$$\frac{d^2y}{dx^2} = -60(x+1)^{-4}$$

$$= \frac{-60}{(x+1)^4}$$

$$\text{so } \frac{d^2y}{dx^2} = -\frac{15}{4} = \frac{-60}{(x+1)^4}$$

$$-15(x+1)^4 = -240$$

$$(x+1)^4 = 16$$

$$x+1 = \pm 2$$

$$x = 1 \text{ and } x = -3$$

3. Rabbits were introduced onto an island. The number of rabbits, P , t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \quad t \in \mathbb{R}, t \geq 0$$

(a) Write down the number of rabbits that were introduced to the island.

(1)

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.

(2)

(c) Find $\frac{dP}{dt}$.

(2)

(d) Find P when $\frac{dP}{dt} = 50$.

(3)

a) $t = 0 \quad P = 80e^0$
 $P = 80$

b) $P = 1000 \Rightarrow 1000 = 80e^{1/5t}$

$$\frac{1000}{80} = e^{1/5t}$$

$$\ln\left(\frac{1000}{80}\right) = 1/5t$$

$$t = 5 \ln\left(\frac{1000}{80}\right)$$

$$t = 12.6 \text{ or } \underline{13 \text{ years.}}$$

c) $\frac{dP}{dt} = 16e^{1/5t}$

d) $50 = 16e^{1/5t}$

$$\frac{50}{16} = e^{1/5t} \Rightarrow \ln\left(\frac{50}{16}\right) = 1/5t \Rightarrow t = 5 \ln\left(\frac{50}{16}\right)$$

$$P = 80e^{1/5(5 \ln(\frac{50}{16}))} \quad * \text{do not round.}$$



4. (i) Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\frac{\ln(x^2 + 1)}{x^2 + 1}$

(4)

(ii) A curve C has the equation

$$y = \sqrt{4x+1}, \quad x > -\frac{1}{4}, \quad y > 0$$

The point P on the curve has x -coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a , b and c are integers.

(6)

(i) (a) $u = x^2 \quad v = \cos 3x$

$$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = -3\sin 3x$$

product rule.

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{dy}{dx} = -3x^2 \sin 3x + 2x \cos 3x$$

b) $u = \ln(x^2 + 1) \quad v = x^2 + 1$ quotient rule

$$\frac{du}{dx} = \frac{2x}{x^2 + 1} \quad \frac{dv}{dx} = 2x$$

$$\frac{dy}{dx} = v \frac{du}{dx} - u \frac{dv}{dx}$$

 v^2

$$= \cancel{(x^2 + 1)} \frac{2x}{\cancel{x^2 + 1}} - \ln(x^2 + 1) \cdot 2x$$

$$\frac{d\cancel{y}}{dx} = \frac{(x^2 + 1)^2}{(x^2 + 1)^2}$$

$$= \frac{2x - 2x \ln(x^2 + 1)}{(x^2 + 1)^2}$$



Question 4 continued

$$y = (4x+1)^{\frac{1}{12}}$$

Gradient $\frac{dy}{dx} = \frac{1}{2} (4x+1)^{-\frac{1}{12}} \cdot 4$

$$= 2(4x+1)^{-\frac{1}{12}}$$

When $x = 2$ $y = (4 \times 2 + 1)^{\frac{1}{12}}$

$$y = 3$$

When $x = 2$ $\frac{dy}{dx} = 2(4 \times 2 + 1)^{-\frac{1}{12}}$

$$(m) \frac{dy}{dx} = \frac{2}{3}$$

Using ' $y = mx + c$ '

$$3 = \frac{2}{3} \cdot 2 + c$$

$$c = \frac{5}{3}$$

$$\therefore y = \frac{2}{3}x + \frac{5}{3} \quad * \text{ Put in form } ax + by + c = 0$$

$$2x - 3y + 5 = 0$$



2. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \quad x \neq \frac{5}{3}.$$

The point P on C has x -coordinate 2.

Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a , b and c are integers.

(7)

$$y = 3(5-3x)^{-2}$$

$$\begin{aligned}\frac{dy}{dx} &= -6(5-3x)^{-3} \cdot -3 \\ &= 18(5-3x)^{-3}\end{aligned}$$

$$x = 2 \quad \frac{dy}{dx} = -18$$

(tangent)

$$\text{Gradient Normal} = \frac{1}{18}$$

$$\text{when } x = 2 \quad y = \frac{3}{(5-3 \times 2)^2}$$

$$y = 3$$

$$y - 3 = \frac{1}{18}(x - 2)$$

$$18y - 54 = x - 2$$

$$x - 18y + 52 = 0$$

1. Differentiate with respect to x

(a) $\ln(x^2 + 3x + 5)$ (2)

(b) $\frac{\cos x}{x^2}$ (3)

a)
$$\frac{2x+3}{x^2+3x+5}$$

b) $u = \cos x \quad v = x^2$
 $\frac{du}{dx} = -\sin x \quad \frac{dv}{dx} = 2x$

Quotient rule

$$\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$= \frac{x^2(-\sin x) - 2x \cos x}{x^4}$$

OR $= \frac{-x^2 \sin x - 2x \cos x}{x^4}$

OR $= \frac{-x \sin x - 2 \cos x}{x^3}$



7. (a) Differentiate with respect to x ,

(i) $x^{\frac{1}{2}} \ln(3x)$

(ii) $\frac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form. (6)

(b) Given that $x = 3 \tan 2y$ find $\frac{dy}{dx}$ in terms of x . (5)

7a) (i) $u = x^{\frac{1}{2}}$ $v = \ln 3x$

$$\begin{aligned}\frac{du}{dx} &= \frac{1}{2}x^{-\frac{1}{2}} & \frac{dv}{dx} &= \frac{3}{3x} \\ &= \frac{1}{x}\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2}x^{-\frac{1}{2}} \ln(3x) + x^{\frac{1}{2}} \left(\frac{1}{x}\right) \\ &= \frac{\ln(3x)}{2x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{2}}}\end{aligned}$$

(ii) Quotient Rule

$$u = 1-10x \quad v = (2x-1)^5$$

$$\frac{du}{dx} = -10 \quad \frac{dv}{dx} = 10(2x-1)^4$$

$$\frac{dy}{dx} = \frac{-10(2x-1)^5 - 10(2x-1)^4(1-10x)}{(2x-1)^{10}}$$

$$= \frac{-10(2x-1)^5 - 10(1-10x)(2x-1)^4}{(2x-1)^{10}}$$



Question 7 continued

$$\begin{aligned}\frac{dy}{dx} &= \frac{-10(2x-1) - 10(1-10x)}{(2x-1)^6} \\ &= \frac{-20x + 10 - 10 + 100x}{(2x-1)^6} \\ &= \frac{80x}{(2x-1)^6}\end{aligned}$$

(b)

$$x = 3 \tan 2y$$

$$\frac{dx}{dy} = 6 \sec^2 2y$$

$$\boxed{\frac{dy}{dx} = \frac{1}{6 \sec^2 2y}}$$

$$\frac{dy}{dx} = \frac{1}{6} (\sec^2 2y)$$

Remember

$$x = 3 \tan 2y$$

$$(\text{square}) \quad x^2 = 9 \tan^2 2y$$

$$\frac{x^2}{9} = \tan^2 2y \quad (\text{but } \tan^2 2y + 1 = \sec^2 2y)$$

$$\text{so } \frac{x^2}{9} = \sec^2 2y - 1$$

$$\text{so } \sec^2 2y = \frac{x^2}{9} + 1$$

substitute into $\frac{dy}{dx}$ 

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Question 7 continued

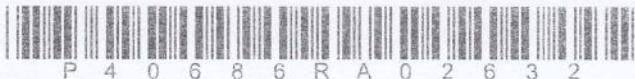
$$\frac{dy}{dx} = \frac{1}{6 \sec^2 2y}$$

$$\frac{dy}{dx} = \frac{1}{6 \left(\frac{x^2}{9} + 1 \right)}$$

$$= \frac{1}{6 \left(\frac{x^2 + 9}{9} \right)}$$

$$= \frac{9}{6(x^2 + 9)}$$

$$\frac{dy}{dx} = \frac{3}{2(x^2 + 9)}$$



1. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

- (a) the value of w ,

(2)

- (b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

a) when $x = w \Rightarrow y = -32$
 $-32 = (2w - 3)^5$

5th root of -32 is -2

$$-2 = 2w - 3$$

$$3 - 2 = 2w$$

$$1 = 2w$$

$$w = \frac{1}{2}$$

$$\text{So } w = \frac{1}{2}$$

b) $y = (2x - 3)^5$
 $\frac{dy}{dx} = 2 \times 5 (2x - 3)^4 = 10 (2x - 3)^4$

at $x = \frac{1}{2}$, $\frac{dy}{dx} = 10 (2 \times \frac{1}{2} - 3)^4$

$$\frac{dy}{dx} = 10 (1 - 3)^4 = 10 (-2)^4 = 160$$

$$y - y_1 = m(x - x_1)$$

$$y - (-32) = 160(x - \frac{1}{2})$$

$$y + 32 = 160x - 80$$

$$y = 160x - 80 - 32$$

$$\underline{\underline{y = 160x - 112}}$$



5. (i) Differentiate with respect to x

(a) $y = x^3 \ln 2x$

(b) $y = (x + \sin 2x)^3$

(6)

Given that $x = \cot y$,

(ii) show that $\frac{dy}{dx} = \frac{-1}{1+x^2}$

(5)

$$\text{(i) a) } y = x^3 \ln 2x \quad u = x^3 \quad v = \ln 2x$$

$$\frac{du}{dx} = 3x^2 \quad \frac{dv}{dx} = \frac{1}{2x}$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx} = x^3 \cdot \frac{1}{2x} + 3x^2 \ln 2x$$

$$\frac{dy}{dx} = x^2 + 3x^2 \ln 2x$$

$$\text{b) } y = (x + \sin 2x)^3$$

$$\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2\cos 2x)$$

$$\text{(ii) } x = \cot y = \frac{\cos y}{\sin y}$$

$$u = \cos y \quad v = \sin y$$

$$\frac{du}{dy} = -\sin y \quad \frac{dv}{dy} = \cos y$$

$$\frac{dx}{dy} = \frac{v \frac{du}{dy} - u \frac{dv}{dy}}{v^2}$$

$$\frac{dx}{dy} = \frac{\sin y \times -\sin y - \cos y \times \cos y}{\sin^2 y}$$

$$\frac{dx}{dy} = \frac{-\sin^2 y - \cos^2 y}{\sin^2 y}$$



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5(ii) (continued)

$$\frac{dx}{dy} = -1 - \frac{\cos^2 y}{\sin^2 y}$$

$$\frac{dx}{dy} = -1 - \cot^2 y$$

$$\frac{dx}{dy} = -1 - x^2$$

$$\frac{dx}{dy} = -(1+x^2)$$

$$\frac{dy}{dx} = \frac{1}{-(1+x^2)}$$

$$\frac{dy}{dx} = \frac{-1}{1+x^2}$$

as required

5. Given that

$$x = \sec^2 3y, \quad 0 < y < \frac{\pi}{6}$$

(a) find $\frac{dx}{dy}$ in terms of y . (2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}} \quad (4)$$

(c) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x . Give your answer in its simplest form. (4)

a) $x = \sec^2 3y = (\sec 3y)^2$

$$\begin{aligned} \frac{dx}{dy} &= 2(\sec 3y)^1 \times 3 \sec 3y \tan 3y \\ &= \underline{6 \sec^2 3y \tan 3y} \end{aligned}$$

b) $\frac{dy}{dx} = \frac{1}{6 \sec^2 3y \tan 3y}$

$$\frac{dy}{dx} = \frac{1}{6x\sqrt{x-1}}$$

$x = \sec^2 3y$
 $\sec^2 3y = 1 + \tan^2 3y$
 $\therefore x = 1 + \tan^2 3y$
 $x - 1 = \tan^2 3y$
 $\sqrt{x-1} = \tan 3y$



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$$5c) \frac{dy}{dx} = \frac{1}{6x\sqrt{x-1}} = \frac{x^{-1} \leftarrow u}{6(x-1)^{\frac{1}{2}} \leftarrow v}$$

$$\frac{d^2y}{dx^2} = \frac{v \frac{dy}{dx} - u \frac{dv}{dx}}{v^2}$$

$$u = x^{-1} \quad v = 6(x-1)^{\frac{1}{2}}$$

$$u' = -1x^{-2} \quad v' = 3(x-1)^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{-6(x-1)^{\frac{1}{2}}x^{-2} - x^{-1} \cdot 3(x-1)^{-\frac{1}{2}}}{6^2 \times ((x-1)^{\frac{1}{2}})^2}$$

$$= \frac{-6(x-1)^{\frac{1}{2}}}{x^2} - \frac{3(x-1)^{-\frac{1}{2}}}{x}$$

$$\underline{36(x-1)}$$

$$= \frac{-6(x-1)^{\frac{1}{2}}}{x^2} - \frac{3}{x(x-1)^{\frac{1}{2}}}$$

$$\underline{36(x-1)}$$

$$= \frac{-6((x-1)^{\frac{1}{2}})^2 - 3x}{x^2(x-1)^{\frac{1}{2}}} = \frac{-6(x-1) - 3x}{36x^2(x-1)^{\frac{3}{2}}}$$

$$\underline{36(x-1)}$$

$$= \frac{-6x + 6 - 3x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{-9x + 6}{36x^2(x-1)^{\frac{3}{2}}}$$

$$= \frac{-8(3x-2)}{12 \cancel{36} x^2 (x-1)^{\frac{3}{2}}} = \frac{-3x+2}{12x^2(x-1)^{\frac{3}{2}}}$$