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6. (a) Use the double angle formulae and the identity

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

(b) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$

$\cos 2x = 2\cos^2 x - 1$

$\sin 2x = 2\sin x \cos x$

(3)

6a) $\cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x$

$= (2\cos^2 x - 1)\cos x - 2\sin^2 x \cos x$

$= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= 4\cos^3 x - 3\cos x.$

b) (i) common denominator

$\cos^2 x + \sin^2 x = 1$

$$\frac{\cos^2 x + (1+\sin x)(1+\sin x)}{(1+\sin x)\cos x} = \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}$$

$$= \frac{2 + 2\sin x}{(1+\sin x)\cos x}$$

$$= \frac{2\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}\cos x}$$

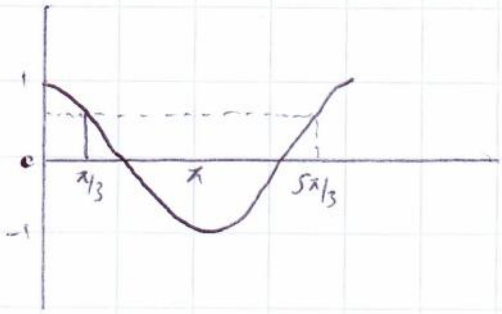
$$= 2\sec x.$$

6b (ii)

$2\sec x = 4$

$\sec x = 2$

$\cos x = \frac{1}{2}$



$x = \pi/3, 5\pi/3$

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7. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \ -\pi \leqslant x \leqslant \pi .$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A.

(5)

(b) Express y in the form $R\sin(2x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis. Give your answers to 2 decimal places.

(4)

7) $\frac{dy}{dx} = 6\cos 2x - 8\sin 2x$

$x = 0$ $\quad \underset{\text{tangent}}{\frac{dy}{dx}} = 6 \qquad \underset{\text{normal}}{\frac{dy}{dx}} = -\frac{1}{6}$

$y - 4 = -\frac{1}{6}x$

b) $R = \sqrt{3^2 + 4^2}$ $\quad R\sin(2x+\alpha) = R\sin 2x\cos\alpha + R\cos 2x\sin\alpha$

$R = 5$

$R\cos\alpha = 3 \qquad R\sin\alpha = 4$

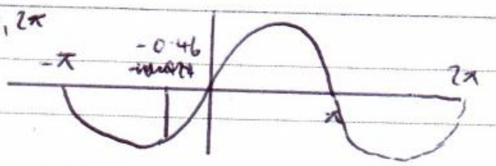
$\frac{\sin\alpha}{\cos\alpha} = \tan\alpha = \frac{4}{3} \Rightarrow \alpha = 0.927$

$\Rightarrow 5\sin(2x + 0.927)$

c) $5\sin(2x + 0.927) = 0$ $\quad$ Intersection of x-axis: $y = 0$.

$\sin(2x + 0.927) = 0$

$2x + 0.927 = 0$, or $-\pi, \pi, 2\pi$



$2x + 0.927 = -\pi \qquad 2x + 0.927 = 2\pi \qquad 2x = -0.927$

$\underline{x = -2.03} \qquad \underline{x = 2.68} \qquad \underline{x = -0.46}$

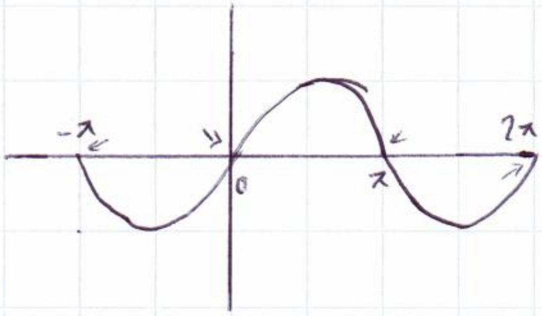
$2x + 0.927 = \pi$

$\underline{x = 1.1}$

Again Clearly!

c) $5\sin(2x + 0.927) = 0$ Intersection of x-axis $\underline{y=0}$

$\sin(2x + 0.927) = 0$



① $2x + 0.927 = 0$

② or $2x + 0.927 = -\pi$

③ or $2x + 0.927 = \pi$

④ or $2x + 0.927 = 2\pi$

① $2x + 0.927 = 0$

$2x = -0.927$

$\underline{x = -0.46}$

② $2x + 0.927 = -\pi$

$2x = -4.068$

$\underline{x = -2.03}$

③ $2x + 0.927 = \pi$

$2x = 2.2$

$\underline{x = 1.1}$

④ $2x + 0.927 = 2\pi$

$2x = 5.36$

$\underline{x = 2.68}$

Leave blank

6. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8\sin^3\theta - 6\sin\theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(b) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

$\sin(A+B) = \sin A\cos B + \sin B\cos A$

$\sin(2\theta+\theta) = \sin 2\theta\cos\theta + \sin\theta\cos 2\theta$

Substitute $\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = (1 - 2\sin^2\theta)$

$\sin(2\theta+\theta) = 2\sin\theta\cos\theta \cdot \cos\theta + \sin\theta(1 - 2\sin^2\theta)$

$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$

Sub. $\cos^2\theta = 1 - \sin^2\theta$

$\sin(2\theta+\theta) = 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$

(ii) Compare $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

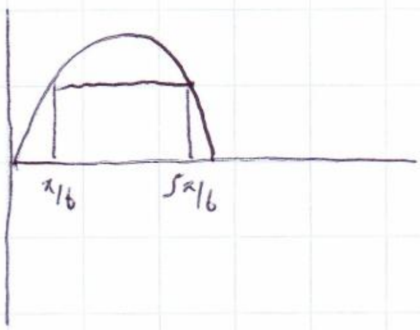
to $8\sin^3\theta - 6\sin\theta + 1 = 0$

Multiply $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ by -2

$-2\sin 3\theta = 8\sin^3\theta - 6\sin\theta$

$\Rightarrow$ $-2\sin 3\theta + 1 = 0$

Limits $0 < \theta < \pi/3$



$\sin 3\theta = \frac{1}{2}$

$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$

b) $\sin 15° = \sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45°$

always split using 60°, 45° or 30°

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$

Leave blank

8. (a) Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of $3\cos\theta + 4\sin\theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3\cos(15t)° + 4\sin(15t)°,$$

where t is the time in hours from midday and $0 \leqslant t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)

a) $R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha \equiv 3\cos\theta + 4\sin\theta$

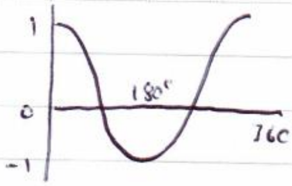
$R\cos\alpha = 3$ and $R\sin\alpha = 4$

$\tan\alpha = \frac{4}{3}$ also $R^2 = 3^2 + 4^2$

$\alpha = 53°$ $R = 5$

$5\cos(\theta - 53°)$

b) Max. Value when $\cos(\theta - 53°) = 1$ Max. Value $= 5$.



$\cos 0° = 1$

$\Rightarrow \theta = 53°$

c) $f(t) = 10 + 3\cos(15t) + 4\sin(15t)$

$= 10 + 5\cos(15t - \alpha)$ (using parts a) b))
$\theta = 15t$.

Minimum when $\cos(15t - \alpha) = -1$
Minimum $= 10 - 5 = \underline{5}$

d) $15t - \alpha = 180°$ $(\alpha = 53°)$ $(\cos^{-1}(-1) = 180°)$
$t = 15.5$

Leave blank

3. (a) Express $5\cos x - 3\sin x$ in the form $R\cos(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(b) Hence, or otherwise, solve the equation

$$5\cos x - 3\sin x = 4$$

for $0 \leqslant x < 2\pi$, giving your answers to 2 decimal places.

(5)

$5\cos x - 3\sin x = R\cos(x+\alpha)$

$= R\cos x\cos\alpha - R\sin x\sin\alpha$

Equate : $\cos x$ $\quad 5 = R\cos\alpha$

: $\sin x$ $\quad 3 = R\sin\alpha$

$R = \sqrt{5^2+3^2}$ $\qquad \dfrac{\sin\alpha}{\cos\alpha} = \tan\alpha = \dfrac{3}{5}$ radians!

$R = \sqrt{34}$

$\alpha = 0.54^c$

Hence $5\cos x - 3\sin x = \sqrt{34}\cos(x + 0.5404)$

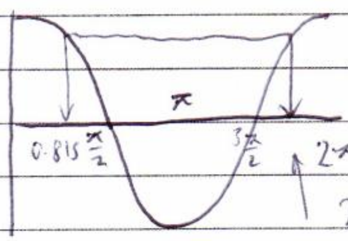
(b) $5\cos x - 3\sin x = 4$

$\sqrt{34}\cos(x + 0.5404) = 4$

$\cos(x + 0.5404) = \dfrac{4}{\sqrt{34}}$

$x + 0.5404 = 0.815$

$\underline{\underline{x = 0.27}}$



$2\pi - 0.815 = 5.468$ so $x + 0.5404 = 5.468$

$\underline{\underline{x = 4.93}}$

Leave blank

8. Solve

$$\text{cosec}^2 2x - \cot 2x = 1$$

for $0 \leqslant x \leqslant 180°$.

(7)

$\text{cosec}^2 2x - \cot 2x = 1$

$(\text{cosec}^2 2x = 1 + \cot^2 2x)$

so : $1 + \cot^2 2x - \cot 2x = 1$

$\cot^2 2x - \cot 2x = 0$

$\cot 2x (\cot 2x - 1) = 0$

$\cot 2x = 0$ or $\cot 2x = 1$

$\Rightarrow \tan 2x \to \infty$ $\Rightarrow \tan 2x = 1$

$2x = 90, 270$ $2x = 45, 225$

$\underline{x = 45, 135}$ $\underline{x = 22.5, 112.5}$

1. (*a*) Express $7 \cos x - 24 \sin x$ in the form $R \cos (x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

(*b*) Hence write down the minimum value of $7 \cos x - 24 \sin x$.

(1)

(*c*) Solve, for $0 \leq x < 2\pi$, the equation

$$7 \cos x - 24 \sin x = 10,$$

giving your answers to 2 decimal places.

(5)

1a) $R^2 = 7^2 + 24^2$

$R = 25$

$R\cos(x+\alpha) = R\cos x\cos\alpha - R\sin x\sin\alpha$

$R\cos x\cos\alpha = 7\cos x \Rightarrow R\cos\alpha = 7$

$R\sin x\sin\alpha = 24\sin x \Rightarrow R\sin\alpha = 24$

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{24}{7}$$

$$\tan\alpha = \frac{24}{7}$$

$\alpha = 1.287$

$\Rightarrow 25\cos(x + 1.287)$

b) Min value $= -25$

c) $7\cos x - 24\sin x = 10$

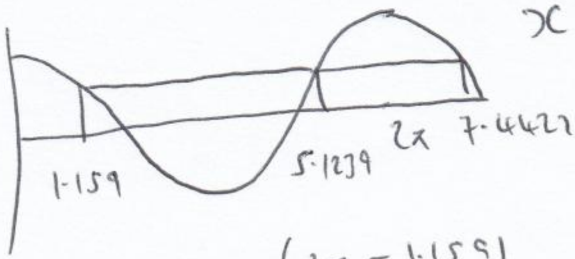
$$25\cos(x + 1.287) = 10$$

$$\cos(x + 1.287) = \frac{10}{25}$$

$$x + 1.287 = \cos^{-1}\left(\frac{10}{25}\right)$$

$$x + 1.287 = \cancel{1.159},\ 5.1239,\ 7.4422$$

$x = \underline{3.84},\ \underline{6.16}$



$(2\pi - 1.159)$

3. Find all the solutions of

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0 \le \theta < 360°$.

(6)

3

$\cos 2\theta = 1 - 2\sin^2\theta$

$2(1 - 2\sin^2\theta) = 1 - 2\sin\theta$

$2 - 4\sin^2\theta = 1 - 2\sin\theta$

$4\sin^2\theta - 2\sin\theta - 1 = 0$

$(\sin\theta + 1)(\sin\theta - 1) = 0$

X

so. Quadratic formula!

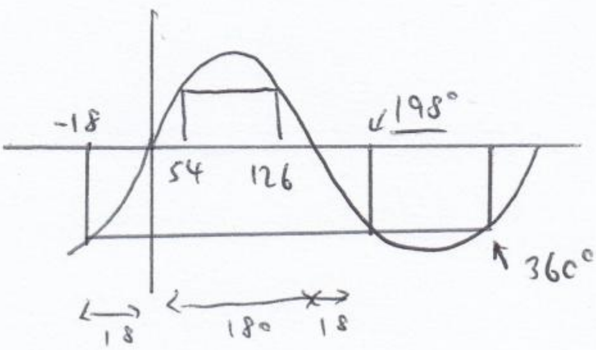
$\sin\theta = \dfrac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$

$\sin\theta = 0.809$

$\theta = 54°$

or $\sin\theta = -0.309$

$\theta = -18°$



$\theta = \underline{54°}, \underline{126°}$

$\theta = \underline{198°}, \underline{342°}$

$360° - 18° = \underline{342°}$

Leave blank

5. Solve, for $0 \leqslant \theta \leqslant 180°$,

$$2\cot^2 3\theta = 7\operatorname{cosec} 3\theta - 5$$

Give your answers in degrees to 1 decimal place.

(10)

$\cot^2 3\theta = \operatorname{cosec}^2 3\theta - 1$

$\Rightarrow \quad 2(\operatorname{cosec}^2 3\theta - 1) = 7\operatorname{cosec} 3\theta - 5$

$\Rightarrow \quad 2\operatorname{cosec}^2 3\theta - 2 = 7\operatorname{cosec} 3\theta - 5$

$\Rightarrow \quad 2\operatorname{cosec}^2 3\theta - 7\operatorname{cosec} 3\theta + 3 = 0$

$(2\operatorname{cosec} 3\theta - 1)(\operatorname{cosec} 3\theta - 3) = 0$

$\operatorname{cosec} 3\theta = \frac{1}{2}$ or $\operatorname{cosec} 3\theta = 3$

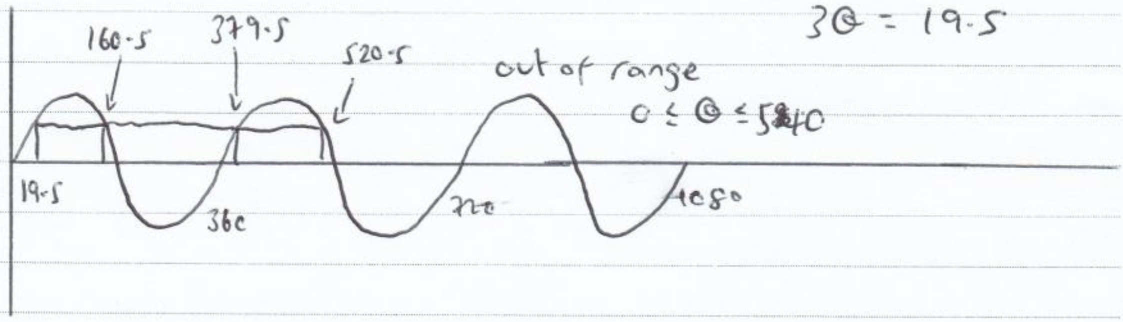
$\sin 3\theta = 2$ $\qquad \sin 3\theta = \frac{1}{3}$

$\times$ impossible

$3\theta = \sin^{-1}(1/3)$

$3\theta = 19.47$

$3\theta = 19.5$



$3\theta = 19.5, 160.5, 379.5, 520.5$

$\theta = 6.5°, 53.5°, 126.5°, 173.5°$

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C3 Jan 2012

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8. (a) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

(4)

(b) Deduce that

$$\tan\left(\theta+\frac{\pi}{6}\right)=\frac{1+\sqrt{3}\tan\theta}{\sqrt{3}-\tan\theta}$$

(3)

(c) Hence, or otherwise, solve, for $0\leqslant\theta\leqslant\pi$,

$$1+\sqrt{3}\tan\theta=(\sqrt{3}-\tan\theta)\tan(\pi-\theta)$$

Give your answers as multiples of π.

(6)

a) $\dfrac{\sin(A+B)}{\cos(A+B)} = \dfrac{\sin A\cos B+\cos A\sin B}{\cos A\cos B-\sin A\sin B}$

$\tan(A+B) = \dfrac{\dfrac{\sin A\cos B}{\cos A\cos B}+\dfrac{\cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B}{\cos A\cos B}-\dfrac{\sin A\sin B}{\cos A\cos B}}$ $(\div\cos A\cos B)$

$=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$

(b) $\left(\tan(\theta+\pi/6)\right) = \dfrac{\tan\theta+\tan\pi/6}{1-\tan\theta\tan\pi/6}$

$A=\theta$

$B=\pi/6$

$\tan\pi/6=\frac{1}{\sqrt{3}}$

$=\dfrac{\tan\theta+\frac{1}{\sqrt{3}}}{1-\tan\theta\frac{1}{\sqrt{3}}}$ $\boxed{\times \text{ by } \sqrt{3}}$

$=\dfrac{\sqrt{3}\tan\theta+1}{\sqrt{3}-\tan\theta}$

C3 Jan 2012

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Question 8 continued

(c) $1+\sqrt{3}\tan\theta = (\sqrt{3}-\tan\theta)\tan(\pi-\theta)$

$$\tan(\pi-\theta) = \frac{1+\sqrt{3}\tan\theta}{\sqrt{3}-\tan\theta}$$

using part (b)

$$\tan(\pi-\theta) = \tan(\theta+\pi/6)$$

$$\Rightarrow \quad \pi-\theta = \theta+\pi/6$$

$$2\theta = 5\pi/6$$

$$\boxed{\theta = \frac{5\pi}{12}}$$

and from tan curve

$$2\pi-\theta = \theta+\pi/6$$

$$2\theta = \frac{11\pi}{6}$$

$$\boxed{\theta = \frac{11\pi}{12}}$$

P 4 0 0 8 4 A 0 2 1 2 4

Turn over

Leave blank

6. (a) Express $3 \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **(4)**

(b) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$. **(2)**

(c) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places. **(5)**

$R = \sqrt{3^2 + 2^2}$ $\quad R\sin(x+\alpha) = R\sin x \cos\alpha + R\sin\alpha\cos x$

$= \sqrt{13}$ $\quad R\cos\alpha = 3$ and $R\sin\alpha = 2$

$\frac{\sin\alpha}{\cos\alpha} = \frac{2}{3}$

$\tan\alpha = \frac{2}{3}$

$\alpha = 0.588$

$\Rightarrow \sqrt{13}\sin(x + 0.588)$

b) Greatest value $(3\sin x + 2\cos x)^4$

$= (\sqrt{13}\sin(x + 0.588))^4$

$= \sqrt{13}^4 = (\sqrt{13})^4$

$= 169$

N 2 6 1 0 9 A 0 1 4 2 4

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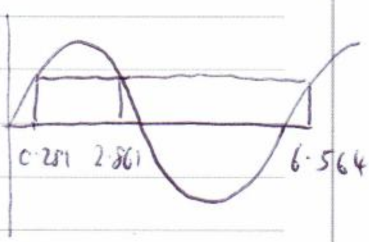
Question 6 continued

(c) $\sin(x + 0.588) = \dfrac{1}{\sqrt{13}}$

$x + 0.588 = 0.281$

and ~~$x=$~~



$x + 0.588 = 2.861$

and $x + 0.588 = 6.564$

$\Rightarrow x = 2.273$ ✓

and $\Rightarrow x = 5.976$ ✓

Leave blank

7. (a) Prove that

$$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=2\operatorname{cosec}2\theta, \qquad \theta\neq 90n°.$$

(4)

(b) On the axes on page 20, sketch the graph of $y = 2\operatorname{cosec}2\theta$ for $0° < \theta < 360°$.

(2)

(c) Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=3,$$

giving your answers to 1 decimal place.

(6)

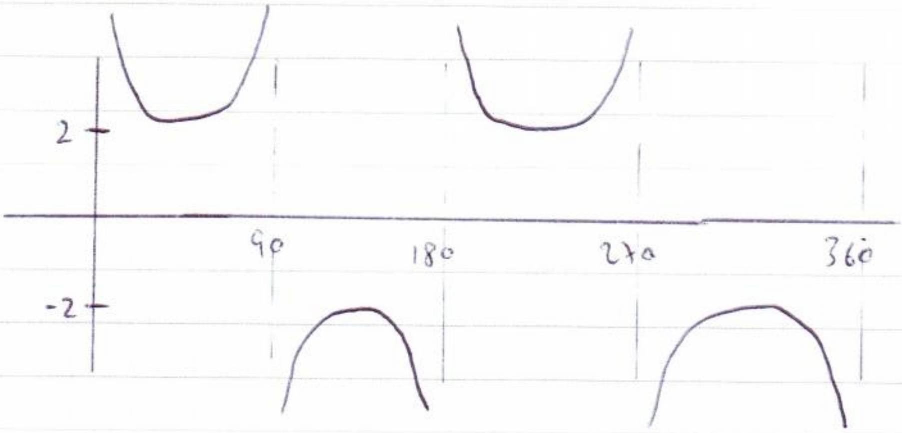
a) $\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta}$

$= \frac{1}{\frac{1}{2}\sin 2\theta}$

$= 2\operatorname{cosec}2\theta.$

b)



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Question 7 continued

e) $2\operatorname{cosec} 2\theta = 3$

$\operatorname{cosec} 2\theta = \frac{3}{2}$

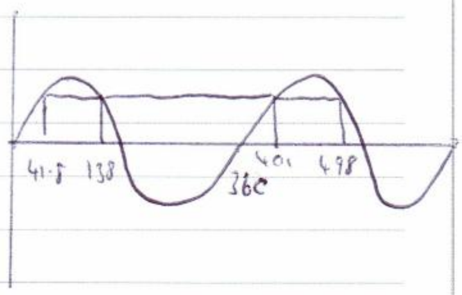
$\sin 2\theta = \frac{2}{3}$

$2\theta = 41.8°$

and $2\theta = 138.2°$

and $2\theta = 401.8°$

and $2\theta = 498.2°$



$\theta = 20.9°, \ 69.1°, \ 200.9°, \ 249.1°$

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Turn over

2.

$$f(x) = 5 \cos x + 12 \sin x.$$

Given that $f(x) = R \cos (x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(*a*) find the value of R and the value of α to 3 decimal places.

(4)

(*b*) Hence solve the equation

$$5 \cos x + 12 \sin x = 6$$

for $0 \leq x < 2\pi$.

(5)

(*c*) (i) Write down the maximum value of $5 \cos x + 12 \sin x$.

(1)

(ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

---

a) $R^2 = 5^2 + 12^2$

$R = 13$

$R\cos(x - \alpha) = R\cos x \cos\alpha + R\sin x \sin\alpha$

$5\cos x \equiv R\cos x \cos\alpha$

$5 = R\cos\alpha$

and $12\sin x \equiv R\sin x \sin\alpha$

$12 = R\sin\alpha$

$\dfrac{R\sin\alpha}{R\cos\alpha} = \dfrac{12}{5}$

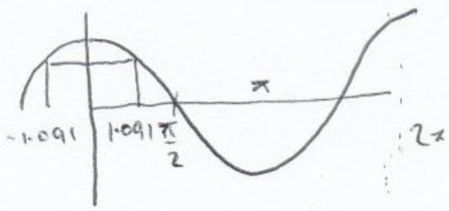
$\tan\alpha = \dfrac{12}{5}$

$\alpha = 1.176$ Radians

(b) $13\cos(x - 1.176) = 6$

$\cos(x - 1.176) = \dfrac{6}{13}$

$x - 1.176 = \cos^{-1}(6/13)$



① $x - 1.176 = 1.091\ldots$

and ② $x - 1.176 = -1.091$

① $x = 2.267$ outside limits. ② $x = 0.085$

c) Max. Value $13\cos(x - 1.176) = 13$

as  $\cos x$

Max. $\cos(x - 1.176) = 1$

(ii) when $\cos(x - 1.176) = 1$

$x - 1.176 = \cos^{-1}(1)$

$x - 1.176 = 0$

$\underline{x = 1.176}$

5. (*a*) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \text{cosec}^2\theta$.

(2)

(*b*) Solve, for $0 \le \theta < 180°$, the equation

$$2\cot^2\theta - 9\,\text{cosec}\,\theta = 3,$$

giving your answers to 1 decimal place.

(6)

(a) $\dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$

$\div$ by $\sin^2\theta$

$1 + \cot^2\theta = \text{cosec}^2\theta$

(b) $2\cot^2\theta - 9\,\text{cosec}\,\theta = 3$

substitute $\cot^2\theta = \text{cosec}^2\theta - 1$

$2(\text{cosec}^2\theta - 1) - 9\,\text{cosec}\,\theta = 3$

$2\,\text{cosec}^2\theta - 2 - 9\,\text{cosec}\,\theta = 3$

$2\,\text{cosec}^2\theta - 9\,\text{cosec}\,\theta - 5 = 0$

$(2\,\text{cosec}\,\theta + 1)(\text{cosec}\,\theta - 5) = 0$

$\text{cosec}\,\theta = -\frac{1}{2}$ and $\text{cosec}\,\theta = 5$

$\sin\theta = -2$ $\quad\quad$ $\sin\theta = \frac{1}{5}$

no soln.

$\theta = 11.5$ and 168.5

$180 - 11.5 = 168.5$

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2. (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$.

(2)

(b) Solve, for $0 \leqslant \theta < 360°$, the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$$

(6)

a) $\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$ $\quad \div$ by $\cos^2\theta$

$\Rightarrow 1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow \tan^2\theta = \sec^2\theta - 1$

(b) Substitute for $\tan^2\theta$

$2(\sec^2(\theta) - 1) + 4\sec\theta + \sec^2\theta = 2$

$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$

$3\sec^2\theta + 4\sec\theta - 4 = 0$

$(3\sec\theta - 2)(\sec\theta + 2) = 0$

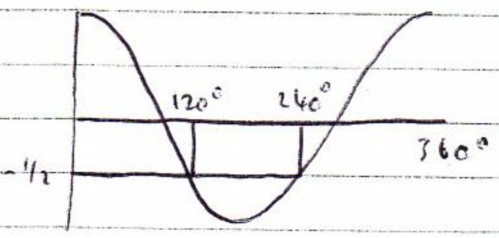
$\sec\theta = \frac{2}{3}$ $\qquad\qquad$ $\sec\theta = -2$

$\Rightarrow \cos\theta = 3/2$ $\qquad\qquad$ $\cos\theta = -\frac{1}{2}$

no solution $\qquad\qquad$ $\underline{\theta = 120°}$, $\underline{240°}$

Use curve

$360° - 120° = \underline{240°}$



H 3 4 2 6 4 A 0 4 2 8

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6. (a) Use the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1: \quad y = 3\sin 2x$$

$$C_2: \quad y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(d) Hence find, for $0 \leqslant x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

(4)

a) Let $A = B$

$$\cos(A+A) = \cos A \cos A - \sin A \sin A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

Remember $\cos^2 A = 1 - \sin^2 A$

$$= (1 - \sin^2 A) - \sin^2 A$$

$$\cos 2A = 1 - 2\sin^2 A$$

(b) Intersect when $C_1 = C_2$

$$3\sin 2x = 4\sin^2 x - 2\cos 2x$$

Eliminate $\sin^2 x$ for substitution $\sin^2 x = \left(\frac{1 - \cos 2x}{2}\right)$

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$$3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$$

$$3\sin 2x = 2(1-\cos 2x) - 2\cos 2x$$

$$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$$

$$3\sin 2x = 2 - 4\cos 2x$$

$$4\cos 2x + 3\sin 2x = 2$$

c)

$$3\sin 2x + 4\cos 2x = R\cos(2x-\alpha)$$

$$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$$

Equating

$$3\cancel{\sin 2x} \equiv R\cancel{\sin 2x}\sin\alpha$$

$$4\cancel{\cos 2x} \equiv R\cancel{\cos 2x}\cos\alpha$$

$R = \sqrt{3^2+4^2}$

$R = 5$

$$\tan\alpha = \frac{3}{4}$$

$$\alpha = 36.87^\circ$$

$$3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87^\circ)$$

Leave blank

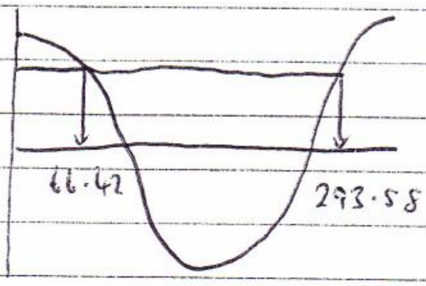
Question 6 continued

d) $4\cos 2x + 3\sin 2x = 2$

$5\cos(2x - 36.87) = 2$

$\cos(2x - 36.87) = \frac{2}{5}$

$2x - 36.87 = 66.42°$



When $2x - 36.87 = 66.42$

$x = 51.65°$

$\underline{x = 51.7°}$

and

$2x - 36.87 = 293.58°$

$\underline{x = 165.2°}$

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8. (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places.

(5)

a) $\sin 2x = 2\sin x \cos x$

(b) $\operatorname{cosec} x - 8\cos x = 0$

$\frac{1}{\sin x} - 8\cos x = 0$ MULTIPLY BY $\sin x$

$1 - 8\sin x \cos x = 0$

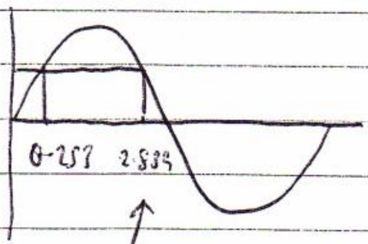
$8\sin x \cos x = 1$

$4(2\sin x \cos x) = 1$

$4(\sin 2x) = 1$

$\sin 2x = 1/4$

Radians $2x = 0.253$



$x = 0.13$

and $2x = 2.889$

$x = 1.44$

2nd soln. $\pi - 0.253$

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1. (*a*) Show that

$$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta.$$

(2)

(*b*) Hence find, for $-180° \le \theta < 180°$, all the solutions of

$$\frac{2\sin 2\theta}{1+\cos 2\theta} = 1.$$

Give your answers to 1 decimal place.

(3)

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

so $\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$

$= \dfrac{\cancel{2}\sin\theta\,\cancel{\cos}\theta}{\cancel{2}\cos^{\cancel{2}}\theta}$

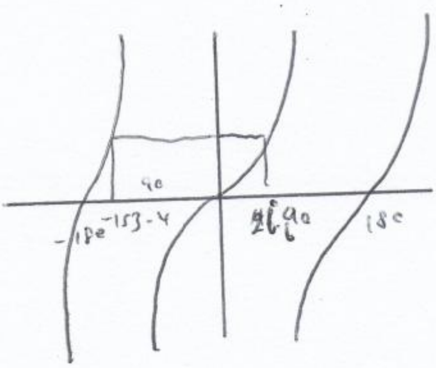
$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta.$

b) $2\tan\theta = 1$

$\cancel{\theta = 45°}$ $\tan\theta = \frac{1}{2}$

$\theta = \underline{26.6°}, \underline{-153.4°}$

$(26.6° - 180°)$



7. (*a*) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

(*b*) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

(ii) Find the value of θ, for $0 \leq \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2 \sin \left(\frac{4\pi t}{25} \right) - 1.5 \cos \left(\frac{4\pi t}{25} \right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

(*c*) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs.

(3)

(*d*) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

---

(7) $R^2 = 2^2 + 1.5^2$

$R = 2.5$

$R\sin(\theta - \alpha) = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$R\sin\theta\cos\alpha = 2\sin\theta$

and $R\cos\theta\sin\alpha = 1.5\cos\theta$

$R\cos\alpha = 2 \qquad R\sin\alpha = 1.5$

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{1.5}{2}$$

$\tan\alpha = 3/4$

$\alpha = 0.6435$

$\Rightarrow 2.5\sin(\theta - 0.6435)$

(b) (i)

$$2\sin\theta - 1.5\cos\theta = 2.5\sin(\theta - 0.6435)$$

MAX. VALUE $= 2.5$

when $\sin(\theta - 0.6435) = 1$

$$\theta - 0.6435 = \frac{\pi}{2}$$

$$\underline{\theta = 2.21}$$

(c)

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right)$$

$$= 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right)$$

MAX $H = 6 + 2.5$

$= 8.5\text{m}$

when $\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1$

$$\frac{4\pi t}{25} - 0.6435 = \frac{\pi}{2}$$

$$\frac{4\pi t}{25} = \frac{\pi}{2} + 0.6435$$

$$t = \frac{25}{4\pi}\left(\pi/2 + 0.6435\right)$$

$$\underline{t = 4.41}$$

(d)

$$6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$$

$$2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1$$

$$\sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5}$$

$$\frac{4\pi t}{25} - 0.6435 = \sin^{-1}\left(\frac{1}{2.5}\right)$$

$$\frac{4\pi t}{25} - 0.6435 = 0.4115$$

$$\frac{4\pi t}{25} = 1.055$$

$$t = 2.1$$

2 hrs

0.1×60 minute

$= 6$ minutes

time $\underline{14.06}$

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2. $$\mathrm{f}(x) = 2\sin(x^2) + x - 2, \qquad 0 \leqslant x < 2\pi$$

(a) Show that $\mathrm{f}(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$

(2)

The equation $\mathrm{f}(x) = 0$ can be written as $x = \left[\arcsin(1 - 0.5x)\right]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1} = \left[\arcsin(1 - 0.5x_n)\right]^{\frac{1}{2}}, \quad x_0 = 0.8$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(c) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

(3)

a) $f(0.75) = -0.18339\ldots$
$f(0.85) = 0.1725$

as there is a change of sign

$0.75 < \alpha < 0.85$

b) REMEMBER arsin is $\sin^{-1}$

$x_0 = 0.8$

$x_1 = \left[\sin^{-1}(1 - 0.5 \times 0.8)\right]^{\frac{1}{2}} = 0.80219$ (5dp)

$x_2 = 0.80133$ (5dp)

$x_3 = 0.80167$ (5dp)

c) $f(0.801565) = 2\sin(0.801565)^2 + x - 2$
$= -2.704 \times 10^{-5}$ < 0

$f(0.801575) = 8.6205 \times 10^{-6}$ > 0

Change of sign
so α between
0.801565 and 0.801575

$\therefore \alpha = 0.80157$ (5dp)

Leave blank

6. (a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \quad \theta \neq 90n°, \ n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1$$

(5)

a) $\dfrac{1}{\sin 2\theta} - \dfrac{\cos 2\theta}{\sin 2\theta} = \dfrac{1 - \cos 2\theta}{\sin 2\theta}$

$\cos 2\theta = 1 - 2\sin^2\theta$
$\sin 2\theta = 2\sin\theta\cos\theta$

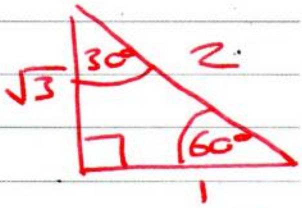
$= \dfrac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta}$

$= \dfrac{\cancel{2}\cancel{\sin\theta}\sin\theta}{\cancel{2}\cancel{\sin\theta}\cos\theta} = \tan\theta$ as required

b)(i) let $\theta = 15$

$\therefore \dfrac{1}{\sin 2\theta} - \dfrac{\cos 2\theta}{\sin 2\theta} = \dfrac{1}{\sin 30°} - \dfrac{1}{\tan 30°}$

$= \dfrac{1}{\frac{1}{2}} - \dfrac{1}{\frac{1}{\sqrt{3}}}$



$\sin 30° = \frac{1}{2}$

$= 2 - \sqrt{3}$ (as required) $\tan 30° = \frac{1}{\sqrt{3}}$

P 3 8 1 5 9 A 0 1 2 2 4

Question 6 continued

(ii) $\operatorname{cosec} 4x - \cot 4x = 1$

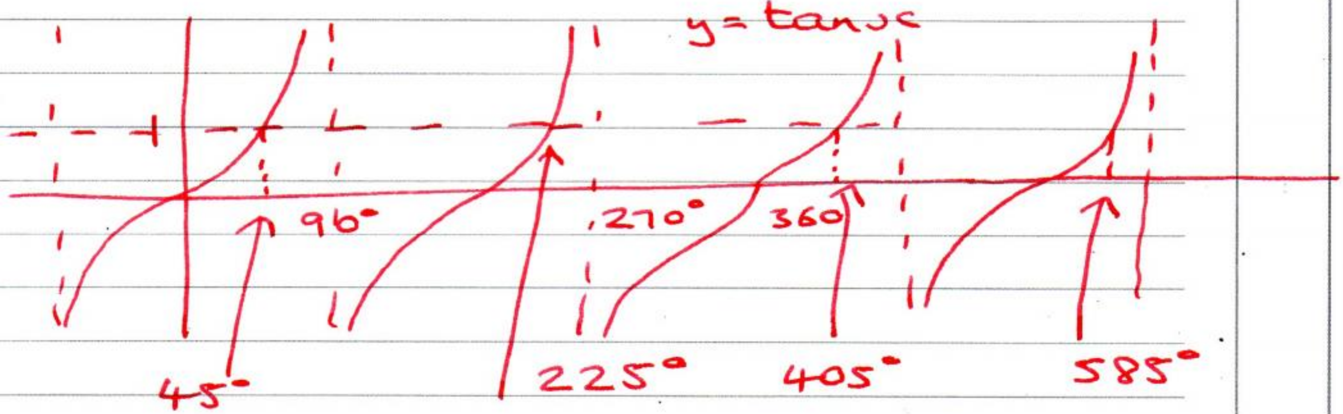
$$\frac{1}{\sin 4x} - \frac{\cos 4x}{\sin 4x} = 1$$

looking at original equation

then $\theta = 2x$

$\therefore \tan 2x = 1$

$2x = \tan^{-1}(1)$



for $0 < x < 360°$

$0 < 2x < 720°$

$\therefore 2x = 45°, 225°, 405°, 585°$

$\therefore x = 22.5°, 112.5°, 202.5°, 292.5°$

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Turn over

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8. (a) Express $2\cos 3x - 3\sin 3x$ in the form $R\cos(3x+\alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures.
(4)

$$f(x) = e^{2x}\cos 3x$$

(b) Show that $f'(x)$ can be written in the form

$$f'(x) = Re^{2x}\cos(3x+\alpha)$$

where R and α are the constants found in part (a).
(5)

(c) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point.
(3)

a) $R\cos(3x+\alpha) = R\cos 3x\cos\alpha - R\sin 3x\sin\alpha$

using double angles

$R\cos 3x\cos\alpha - R\sin 3x\sin\alpha$
$2\cos 3x \quad - 3\sin 3x$

using above $R\cos\alpha = 2$
$R\sin\alpha = 3$

$\therefore \dfrac{R\sin\alpha}{R\cos\alpha} = \tan\alpha = \dfrac{3}{2}$

$\therefore \alpha = \tan^{-1}\left(\frac{3}{2}\right)$
$\alpha = 0.9827937$
$\alpha = 0.983^c$ (3sf)

$R = \sqrt{3^2 + 2^2} = \sqrt{13} = 3.60555$
$R = 3.61$ (3sf)

P 3 8 1 5 9 A 0 2 0 2 4

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Question 8 continued

b) $f(x) = \underset{u}{e^{2x}} \, \underset{v}{\cos 3x}$

$\frac{du}{dx} = 2e^{2x}$ $\qquad$ $\frac{dv}{dx} = -3\sin 3x$

$f'(x) = u\frac{dv}{dx} + v\frac{du}{dx}$

$f'(x) = e^{2x} \cdot (-3\sin 3x) + 2e^{2x}\cos 3x$

$= e^{2x}(-3\sin 3x + 2\cos 3x)$

$= e^{2x}(\underbrace{2\cos 3x - 3\sin 3x}_{\uparrow})$

in part a) we worked out

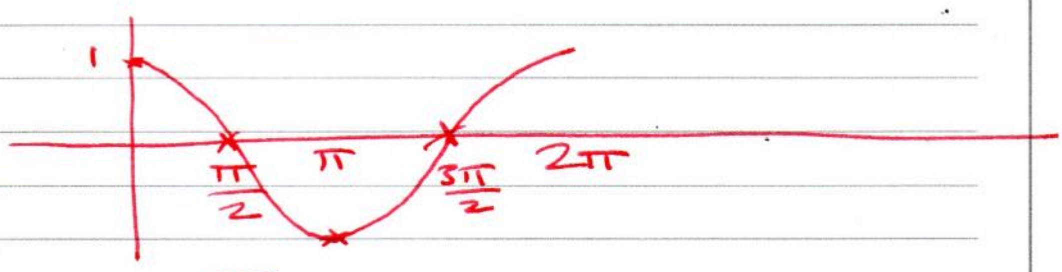
$= e^{2x}(\sqrt{13}\cos(3x + 0.983))$

c) We have turning point when $f'(x) = 0$

$0 = e^{2x} \times \sqrt{13} \times \cos(3x + 0.983)$

$\sqrt{13}\, e^{2x}$ cannot $= 0$

$\therefore \cos(3x + 0.983) = 0$



$0 < \alpha < \frac{\pi}{2}$

$0 < 3x + 0.983 < \frac{3\pi}{2} + 0.983$

Solutions are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

P 3 8 1 5 9 A 0 2 1 2 4