

6. (a) Use the double angle formulae and the identity

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

- (b) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2 \sec x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

- (ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$

$\cos 2x = 2\cos^2 x - 1$
 $\sin 2x = 2\sin x \cos x$

(3)

6(a) $\cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x$

$$= (2\cos^2 x - 1)\cos x - 2\sin^2 x \cos x$$

$$= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$$

$$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$$

$$= 4\cos^3 x - 3\cos x.$$

- b) (i) common denominator

$$\frac{\cos^2 x + (1+\sin x)(1+\sin x)}{(1+\sin x)\cos x} = \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}$$

$$= \frac{2 + 2\sin x}{(1+\sin x)\cos x}$$

$$= \frac{2(1 + \sin x)}{(1 + \sin x)\cos x}$$

$$= 2 \sec x.$$



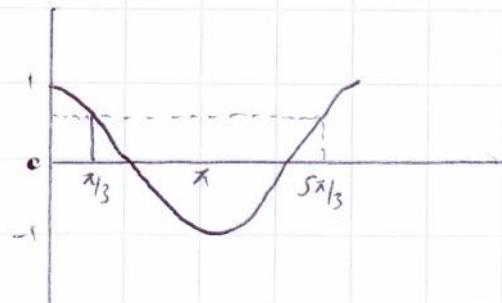
6b (ii)

$$2 \sec x = 4$$

$$\sec x = 2$$

$$\cos x = \frac{1}{2}$$

$$x = \pi/3, 5\pi/3$$



7. A curve C has equation

Leave blank

$$y = 3 \sin 2x + 4 \cos 2x, -\pi \leq x \leq \pi.$$

The point $A(0, 4)$ lies on C .

- (a) Find an equation of the normal to the curve C at A .

(5)

- (b) Express y in the form $R \sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

- (c) Find the coordinates of the points of intersection of the curve C with the x -axis.
Give your answers to 2 decimal places.

(4)

7)
$$\frac{dy}{dx} = 6 \cos 2x - 8 \sin 2x$$

$$x=0 \quad \frac{dy}{dx} = 6$$

tangent

$$\frac{dy}{dx} = -\frac{1}{6}$$

normal

$$y - 4 = -\frac{1}{6}x$$

b) $R = \sqrt{3^2 + 4^2}$ $R \sin(2x + \alpha) = R \sin 2x \cos \alpha + R \cos 2x \sin \alpha$
 $R = 5$

$$R \cos \alpha = 3 \quad R \sin \alpha = 4$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \frac{4}{3} \Rightarrow \alpha = 0.927$$

$$\Rightarrow 5 \sin(2x + 0.927)$$

c) $5 \sin(2x + 0.927) = 0$ Intersection of x -axis: $y = 0$.
 $\sin(2x + 0.927) = 0$

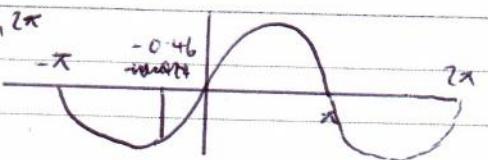
$$2x + 0.927 = 0, \pi, 2\pi$$

$$2x + 0.927 = \pi \Rightarrow x = -0.927$$

$$2x + 0.927 = 2\pi \Rightarrow x = -0.46$$

$$2x + 0.927 = \pi \Rightarrow x = -0.46$$

18 $2x + 0.927 = \pi$
 $x = 1.1$

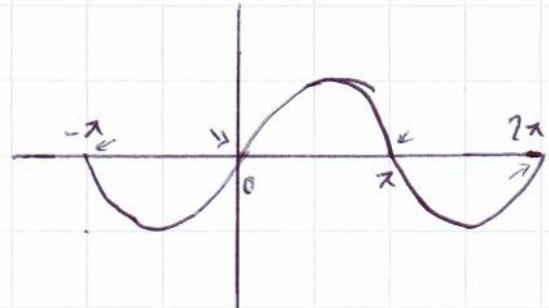


Again Clearly!

c) $5 \sin(2x + 0.927) = 0$ Intersection of x -axis $y=0$

$$\sin(2x + 0.927) = 0$$

① $2x + 0.927 = 0$



② or $2x + 0.927 = -\pi$

③ or $2x + 0.927 = \pi$

④ or $2x + 0.927 = 2\pi$

① $2x + 0.927 = 0$

$$2x = -0.927$$

$$\underline{x = -0.46}$$

② $2x + 0.927 = -\pi$

$$2x = -4.068$$

$$\underline{x = -2.03}$$

③ $2x + 0.927 = \pi$

$$2x = 2.2$$

$$\underline{x = 1.1}$$

④ $2x + 0.927 = 2\pi$

$$2x = 5.36$$

$$\underline{x = 2.68}$$

6. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.$$

(4)

- (ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3 \theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π .

(5)

- (b) Using $\sin(\theta - \alpha) = \sin \theta \cos \alpha - \cos \theta \sin \alpha$, or otherwise, show that

$$\sin 15^\circ = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

$$\sin(A + B) = \sin A \cos B + \sin B \cos A$$

$$\sin(2\theta + \epsilon) = \sin 2\theta \cos \epsilon + \sin \epsilon \cos 2\theta$$

$$\text{Substitute } \sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = (1 - 2 \sin^2 \theta)$$

$$\sin(2\theta + \epsilon) = 2 \sin \theta \cos \theta \cdot \cos \epsilon + \sin \epsilon (1 - 2 \sin^2 \theta)$$

$$= 2 \sin \theta \cos^2 \theta + \sin \epsilon - 2 \sin^3 \theta$$

$$\text{Sub. } \cos^2 \theta = 1 - \sin^2 \theta$$

$$\sin(2\theta + \epsilon) = 2 \sin \epsilon (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta$$

$$= 2 \sin \epsilon - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta$$

$$= 3 \sin \theta - 4 \sin^3 \theta$$



(ii) Compare $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

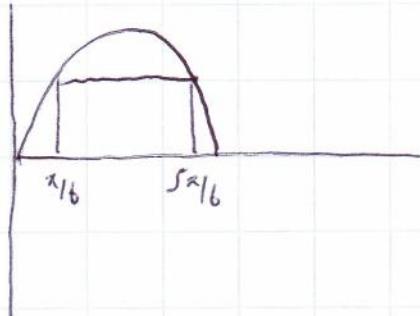
to $8\sin^3\theta - 6\sin\theta + 1 = 0$

Multiply $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ by -2

~~$$-2\sin 3\theta = 8\sin^3\theta - 6\sin\theta$$~~

~~$$\Rightarrow -2\sin 3\theta + 1 = 0$$~~

Limits $0 < \theta < \frac{\pi}{3}$



$$\sin 3\theta = \frac{1}{2}$$

$$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$$

b) $\sin 15^\circ = \sin (60^\circ - 45^\circ) = \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$

always split using
 $60^\circ, 45^\circ, - 30^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{4} (\sqrt{6} - \sqrt{2})$$

8. (a) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90^\circ$.

(4)

- (b) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos(15t)^\circ + 4 \sin(15t)^\circ,$$

where t is the time in hours from midday and $0 \leq t < 24$.

- (c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

- (d) Find the value of t when this minimum temperature occurs.

(3)

$$a) R \cos(\theta - \alpha) = R \cos \theta \cos \alpha + R \sin \theta \sin \alpha \equiv 3 \cos \theta + 4 \sin \theta$$

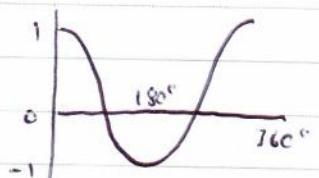
$$R \cos \alpha = 3 \quad \text{and} \quad R \sin \alpha = 4$$

$$\tan \alpha = \frac{4}{3} \quad \text{also} \quad R^2 = 3^2 + 4^2$$

$$\alpha = 53^\circ \quad R = 5$$

$$5 \cos(\theta - 53^\circ)$$

$$b) \text{Max. Value when } \cos(\theta - 53^\circ) = 1 \quad \text{Max. Value} = 5.$$



$$\cos 0^\circ = 1$$

$$\Rightarrow \theta = 53^\circ$$

$$c) f(t) = 10 + 3 \cos(15t) + 4 \sin(15t)$$

$$= 10 + 5 \cos(15t - \alpha) \quad (\text{using part a)(b)})$$

$$\alpha = 53^\circ$$

$$\text{Minimum when } \cos(15t - \alpha) = -1$$

$$\text{Minimum} = 10 - 5 = 5$$

$$d) 15t - \alpha = 180^\circ \quad (\alpha = 53^\circ) \quad (\cos^{-1}(-1) = 180^\circ)$$

$$t = 15.5$$



3. (a) Express $5 \cos x - 3 \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. (4)

(b) Hence, or otherwise, solve the equation

$$5 \cos x - 3 \sin x = 4$$

for $0 \leq x < 2\pi$, giving your answers to 2 decimal places. (5)

$$5 \cos x - 3 \sin x = R \cos(x + \alpha)$$

$$= R \cos x \cos \alpha - R \sin x \sin \alpha$$

Equate : $\cos x \quad 5 = R \cos \alpha$

: $\sin x \quad 3 = R \sin \alpha$

$$R = \sqrt{5^2 + 3^2}$$

$$R = \sqrt{34}$$

$$\frac{\sin \alpha}{\cos \alpha} = \tan \alpha = \frac{3}{5}$$

radians!

$$\alpha = 0.54^\circ$$

Hence $5 \cos x - 3 \sin x = \sqrt{34} \cos(x + 0.5404)$

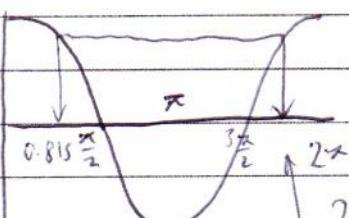
(b) $5 \cos x - 3 \sin x = 4$

$$\sqrt{34} \cos(x + 0.5404) = 4$$

$$\cos(x + 0.5404) = \frac{4}{\sqrt{34}}$$

$$x + 0.5404 = 0.815$$

$$x = 0.27$$



$$2\pi - 0.815 = 5.468 \text{ so } x + 0.5404 = 5.468$$

$$x = 4.93$$

8. Solve

$$\operatorname{cosec}^2 2x - \cot 2x = 1$$

for $0 \leq x \leq 180^\circ$.

(7)

$$\operatorname{cosec}^2 2x - \cot 2x = 1$$

$$(\operatorname{cosec}^2 2x = 1 + \cot^2 2x)$$

$$\text{so : } 1 + \cot^2 2x - \cot 2x = 1$$

$$\cot^2 2x - \cot 2x = 0$$

$$\cot 2x (\cot 2x - 1) = 0$$

$$\cot 2x = 0 \quad \text{or} \quad \cot 2x = 1$$

$$\Rightarrow \tan 2x \rightarrow \infty$$

$$\Rightarrow \tan 2x = 1$$

$$2x = 90, 270$$

$$2x = 45, 225$$

$$x = 45, 135$$

$$x = 22.5, 112.5$$

1. (a) Express $7 \cos x - 24 \sin x$ in the form $R \cos(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(3)

- (b) Hence write down the minimum value of $7 \cos x - 24 \sin x$.

(1)

- (c) Solve, for $0 \leq x < 2\pi$, the equation

$$7 \cos x - 24 \sin x = 10,$$

giving your answers to 2 decimal places.

(5)

1a) $R^2 = 7^2 + 24^2$

$$R = 25$$

$$R \cos(x + \alpha) = R \cos x \cos \alpha - R \sin x \sin \alpha$$

$$R \cos x \cos \alpha = 7 \cos x \Rightarrow R \cos \alpha = 7$$

$$R \sin x \sin \alpha = 24 \sin x \Rightarrow R \sin \alpha = 24$$

$$\frac{R \sin \alpha}{R \cos \alpha} = \frac{24}{7}$$

$$\tan \alpha = \frac{24}{7}$$

$$\alpha = 1.287$$

$$\Rightarrow 25 \cos(x + 1.287)$$

b) min value = -25



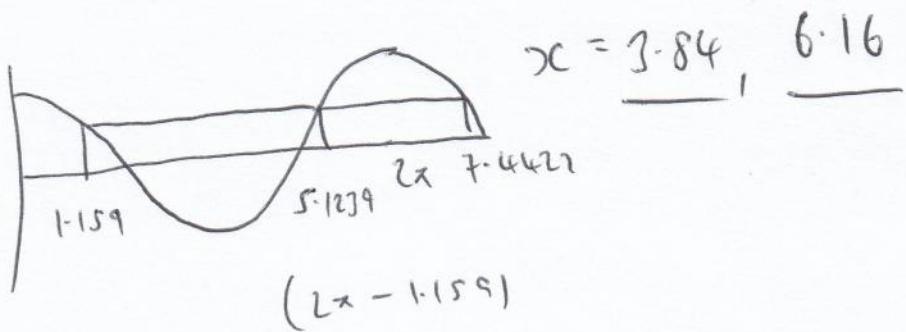
$$c) 7 \cos x - 24 \sin x = 10$$

$$25 \cos(x + 1.287) = 10$$

$$\cos(x + 1.287) = \frac{10}{25}$$

$$x + 1.287 = \cos^{-1}\left(\frac{10}{25}\right)$$

$$x + 1.287 = 1.159, 5.1239, 7.4422$$



3. Find all the solutions of

$$2 \cos 2\theta = 1 - 2 \sin \theta$$

in the interval $0 \leq \theta < 360^\circ$.

(6)

$$\textcircled{3} \quad \cos 2\theta = 1 - 2 \sin^2 \theta$$

$$2(1 - 2 \sin^2 \theta) = 1 - 2 \sin \theta$$

$$2 - 4 \sin^2 \theta = 1 - 2 \sin \theta$$

$$4 \sin^2 \theta - 2 \sin \theta - 1 = 0$$

$$(\sin \theta + 1)(\sin \theta - 1) = 0$$

X

∴ Quadratic formula!

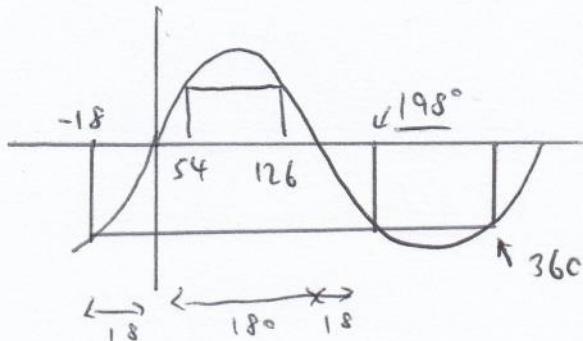
$$\sin \theta = \frac{2 \pm \sqrt{4 - 4(4)(1)}}{8}$$

$$\sin \theta = 0.809$$

$$\theta = 54^\circ$$

$$\text{or } \sin \theta = -0.809$$

$$\theta = -18^\circ$$



$$\theta = 54^\circ, 126^\circ$$

$$\theta = 198^\circ, 342^\circ$$

$$360^\circ - 18^\circ = 342^\circ$$

5. Solve, for $0 \leq \theta \leq 180^\circ$,

$$2\cot^2 3\theta = 7\cosec 3\theta - 5$$

Give your answers in degrees to 1 decimal place.

(10)

$$\cot^2 3\theta = \cosec^2 3\theta - 1$$

$$\Rightarrow 2(\cosec^2 3\theta - 1) = 7\cosec 3\theta - 5$$

$$\Rightarrow 2\cosec^2 3\theta - 2 = 7\cosec 3\theta - 5$$

$$\Rightarrow 2\cosec^2 3\theta - 7\cosec 3\theta + 3 = 0$$

$$(2\cosec 3\theta - 1)(\cosec 3\theta - 3) = 0$$

$$\cosec 3\theta = \frac{1}{2} \quad \text{or} \quad \cosec 3\theta = 3$$

$$\sin 3\theta = 2$$

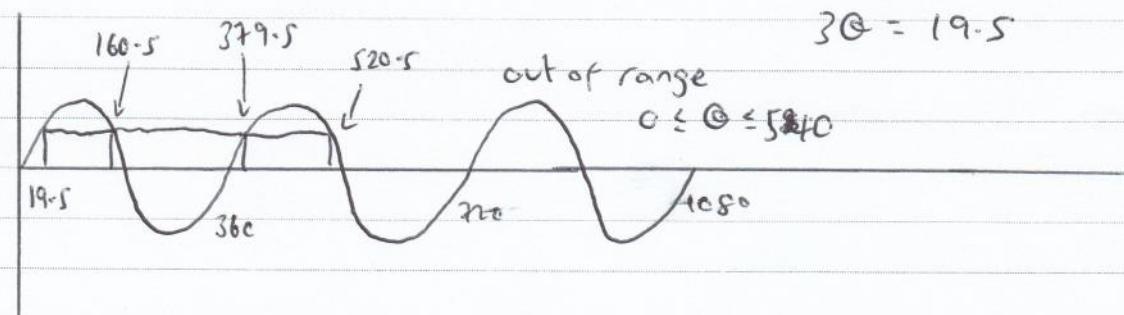
\times impossible

$$\sin 3\theta = \frac{1}{3}$$

$$3\theta = \sin^{-1}\left(\frac{1}{3}\right)$$

$$3\theta = 19.47$$

$$3\theta = 19.5$$



$$3\theta = 19.5, 160.5, 379.5, 520.5$$

$$\theta = 6.5^\circ, 53.5^\circ, 126.5^\circ, 173.5^\circ$$



8. (a) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad (4)$$

- (b) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta} \quad (3)$$

- (c) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3} \tan \theta = (\sqrt{3} - \tan \theta) \tan(\pi - \theta)$$

Give your answers as multiples of π .

(6)

$$\text{a) } \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

$$\tan(A+B) = \frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B} \quad (\div \cos A \cos B)$$

$$\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}$$

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{b) } (\tan(\theta + \pi/6)) = \frac{\tan \theta + \tan \pi/6}{1 - \tan \theta \tan \pi/6}$$

$$A = \theta$$

$$B = \pi/6$$

$$\tan \pi/6 = \frac{1}{\sqrt{3}}$$

$$= \frac{\tan \theta + \frac{1}{\sqrt{3}}}{1 - \tan \theta \frac{1}{\sqrt{3}}}$$

$\times \text{ by } \sqrt{3}$

$$= \frac{\sqrt{3} \tan \theta + 1}{\sqrt{3} - \tan \theta}$$



Question 8 continued

$$(c) \quad 1 + \sqrt{3} \tan \theta = (\sqrt{3} - \tan \theta) \tan(\pi - \theta)$$

$$\tan(\pi - \theta) = \frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta}$$

using part (b)

$$\tan(\pi - \theta) = \tan(\theta + \pi/6)$$

$$\Rightarrow \pi - \theta = \theta + \pi/6$$

$$2\theta = 5\pi/6$$

$$\boxed{\theta = \frac{5\pi}{12}}$$

and from tan curve

$$2\pi - \theta = \theta + \pi/6$$

$$2\theta = 11\pi/6$$

$$\boxed{\theta = \frac{11\pi}{12}}$$



6. (a) Express $3 \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. (4)

(b) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$. (2)

(c) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places. (5)

$$R = \sqrt{3^2 + 2^2}$$

$$R \sin(x + \alpha) = R \sin x \cos \alpha + R \cos x \sin \alpha$$

$$= \sqrt{13}$$

$$R \cos \alpha = 3 \quad \text{and} \quad R \sin \alpha = 2$$

$$\frac{\sin \alpha}{\cos \alpha} = \frac{2}{3}$$

$$\tan \alpha = \frac{2}{3}$$

$$\alpha = 0.588$$

$$\Rightarrow \sqrt{13} \sin(x + 0.588)$$

b) Greatest value $(3 \sin x + 2 \cos x)^4$

$$= (\sqrt{13} \sin(x + 0.588))^4$$

$$= \sqrt{13}^4 = (\sqrt{13})^4$$

$$= 169$$



Question 6 continued

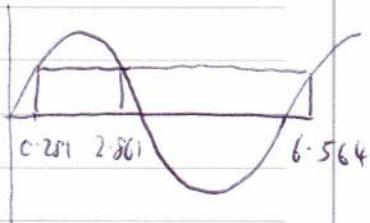
$$(i) \sin(x + 0.588) = \frac{1}{\sqrt{3}}$$

$$x + 0.588 = 0.281$$

and

x14

$$x + 0.588 = 2.861$$



$$\text{and } x + 0.588 = 6.564$$

$$\Rightarrow x = 2.273 \quad \checkmark$$

$$\text{and } \Rightarrow x = 5.976 \quad \checkmark$$



7. (a) Prove that

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 2 \operatorname{cosec} 2\theta, \quad \theta \neq 90n^\circ.$$

(4)

(b) On the axes on page 20, sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0^\circ < \theta < 360^\circ$.

(2)

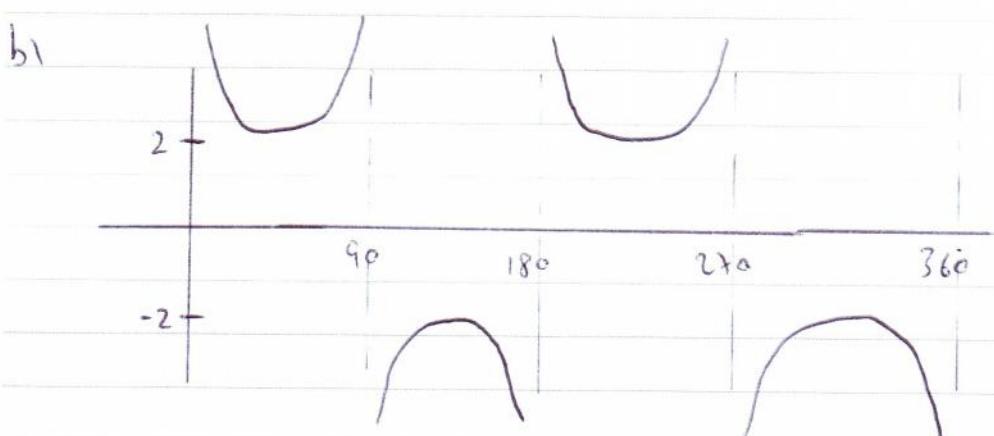
(c) Solve, for $0^\circ < \theta < 360^\circ$, the equation

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = 3,$$

giving your answers to 1 decimal place.

(6)

$$\begin{aligned}
 \text{a) } \frac{\sin \theta}{\operatorname{cosec} \theta} + \frac{\cos \theta}{\sin \theta} &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \operatorname{cosec} \theta} \\
 &= \frac{1}{\sin \theta \operatorname{cosec} \theta} \\
 &= \frac{1}{\frac{1}{2} \sin 2\theta} \\
 &= 2 \operatorname{cosec} 2\theta.
 \end{aligned}$$



Question 7 continued

$$\text{e)} \quad 2\csc 2\theta = 3$$

$$\csc 2\theta = \frac{3}{2}$$

$$\sin 2\theta = \frac{2}{3}$$

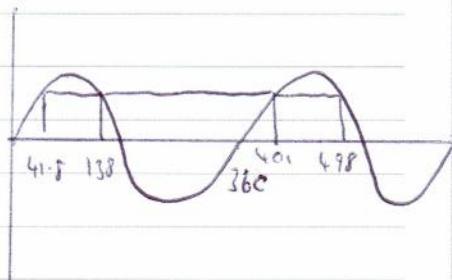
$$2\theta = 41.8^\circ$$

$$\text{and } 2\theta = 138.2^\circ$$

$$\text{and } 2\theta = 401.8^\circ$$

$$\text{and } 2\theta = 498.2^\circ$$

$$\theta = 20.9^\circ, 69.1^\circ, 200.9^\circ, 249.1^\circ$$



2.

$$f(x) = 5 \cos x + 12 \sin x.$$

Given that $f(x) = R \cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(a) find the value of R and the value of α to 3 decimal places. (4)

(b) Hence solve the equation

$$5 \cos x + 12 \sin x = 6$$

for $0 \leq x < 2\pi$. (5)

(c) (i) Write down the maximum value of $5 \cos x + 12 \sin x$. (1)

(ii) Find the smallest positive value of x for which this maximum value occurs. (2)

$$\text{a) } R^2 = 5^2 + 12^2$$

$$R = 13$$

$$R \cos(x - \alpha) = R \cos x \cos \alpha + R \sin x \sin \alpha$$

$$5 \cos x \equiv R \cos x \cos \alpha$$

$$5 = R \cos \alpha$$

$$\text{and } 12 \sin x \equiv R \sin x \sin \alpha$$

$$12 = R \sin \alpha$$

$$\frac{R \sin \alpha}{R \cos \alpha} = \frac{12}{5}$$

$$\tan \alpha = \frac{12}{5}$$

$$\alpha = 1.176 \text{ Radians}$$

$$\text{(b) } 13 \cos(x - 1.176) = 6$$

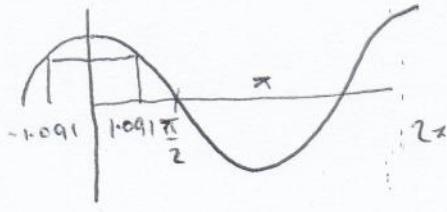
$$\cos(x - 1.176) = \frac{6}{13}$$

$$x - 1.176 = \cos^{-1}\left(\frac{6}{13}\right)$$

$$\textcircled{1} \quad x - 1.176 = 1.091 \dots$$

$$\text{and } \textcircled{2} \quad x - 1.176 = -1.091$$

$$\textcircled{1} \quad x = 2.267 \text{ outside limits.}$$



$$\textcircled{2} \quad x = 0.085$$

c) Max. Value $13 \cos(x - 1.176) = 13$



max. $\cos(x - 1.176) = 1$

(ii) when

$$\cos(x - 1.176) = 1$$

$$x - 1.176 = \cos^{-1}(1)$$

$$x - 1.176 = 0$$

$$\underline{x = 1.176}$$

5. (a) Given that $\sin^2 \theta + \cos^2 \theta \equiv 1$, show that $1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta$.

(2)

- (b) Solve, for $0^\circ \leq \theta < 180^\circ$, the equation

$$2 \cot^2 \theta - 9 \operatorname{cosec} \theta = 3,$$

giving your answers to 1 decimal place.

(6)

$$\begin{aligned} \text{(a)} \quad & \frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta} \\ \div \text{ by } \sin^2 \theta \quad & 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \end{aligned}$$

$$\text{(b)} \quad 2 \cot^2 \theta - 9 \operatorname{cosec} \theta = 3$$

$$\text{substitute } \cot^2 \theta = \operatorname{cosec}^2 \theta - 1$$

$$2(\operatorname{cosec}^2 \theta - 1) - 9 \operatorname{cosec} \theta = 3$$

$$2 \operatorname{cosec}^2 \theta - 2 - 9 \operatorname{cosec} \theta = 3$$

$$2 \operatorname{cosec}^2 \theta - 9 \operatorname{cosec} \theta - 5 = 0$$

$$(2 \operatorname{cosec} \theta + 1)(\operatorname{cosec} \theta - 5) = 0$$

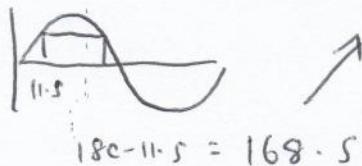
$$\operatorname{cosec} \theta = -\frac{1}{2} \quad \text{and} \quad \operatorname{cosec} \theta = 5$$

$$\sin \theta = -2$$

$$\sin \theta = \frac{1}{5}$$

- no soln.

$$\theta = 11.5^\circ \text{ and } 168.5^\circ$$



$$180^\circ - 11.5^\circ = 168.5^\circ$$

2. (a) Use the identity $\cos^2 \theta + \sin^2 \theta = 1$ to prove that $\tan^2 \theta = \sec^2 \theta - 1$.

(2)

- (b) Solve, for $0^\circ \leq \theta < 360^\circ$, the equation

$$2 \tan^2 \theta + 4 \sec \theta + \sec^2 \theta = 2$$

(6)

a) $\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} \quad \div \text{ by } \cos^2 \theta$

$$\Rightarrow 1 + \tan^2 \theta = \sec^2 \theta$$

$$\Rightarrow \tan^2 \theta = \sec^2 \theta - 1$$

(b) Substitute for $\tan^2 \theta$

$$2(\sec^2 \theta - 1) + 4 \sec \theta + \sec^2 \theta = 2$$

$$2 \sec^2 \theta - 2 + 4 \sec \theta + \sec^2 \theta = 2$$

$$3 \sec^2 \theta + 4 \sec \theta - 4 = 0$$

$$(3 \sec \theta - 2)(\sec \theta + 2) = 0$$

$$\sec \theta = \frac{2}{3}$$

$$\sec \theta = -2$$

$$\Rightarrow \cos \theta = \frac{3}{2}$$

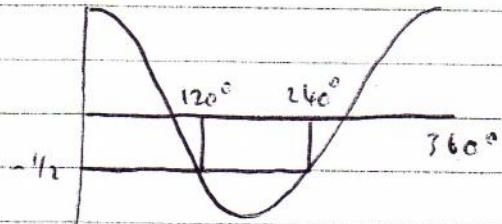
$$\cos \theta = -\frac{1}{2}$$

no solution

$$\theta = 120^\circ, 240^\circ$$

Use curve

$$360^\circ - 120^\circ = 240^\circ$$



6. (a) Use the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2 \sin^2 A \quad (2)$$

The curves C_1 and C_2 have equations

$$C_1: y = 3 \sin 2x$$

$$C_2: y = 4 \sin^2 x - 2 \cos 2x$$

- (b) Show that the x -coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4 \cos 2x + 3 \sin 2x = 2 \quad (3)$$

- (c) Express $4 \cos 2x + 3 \sin 2x$ in the form $R \cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places.

(3)

- (d) Hence find, for $0 \leq x < 180^\circ$, all the solutions of

$$4 \cos 2x + 3 \sin 2x = 2$$

giving your answers to 1 decimal place.

(4)

a) Let $A = B$

$$\cos(A+A) = \cos A \cos A - \sin A \sin A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

Remember $\cos^2 A = 1 - \sin^2 A$

$$\begin{aligned} \cos 2A &= (1 - \sin^2 A) - \sin^2 A \\ &= 1 - 2 \sin^2 A \end{aligned}$$

(b) Intersect when $C_1 = C_2$

$$3 \sin 2x = 4 \sin^2 x - 2 \cos 2x$$

Eliminate $\sin 2x$ for substitution $\sin^2 x = \left(1 - \frac{\cos 2x}{2}\right)$



$$3\sin 2x = 4 \left(\frac{1 - \cos 2x}{2} \right) - 2\cos 2x$$

$$3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$$

$$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$$

$$3\sin 2x = 2 - 4\cos 2x$$

$$4\cos 2x + 3\sin 2x = 2$$

c)

$$3\sin 2x + 4\cos 2x = R \cos(2x - \alpha)$$

$$3\sin 2x + 4\cos 2x = R \cos 2x \cos \alpha + R \sin 2x \sin \alpha$$

Equating:

$$3\sin 2x = R \sin 2x \sin \alpha$$

$$4\cos 2x = R \cos 2x \cos \alpha$$

$$R = \sqrt{3^2 + 4^2}$$

$$R = 5$$

$$\tan \alpha = \frac{3}{4}$$

$$\alpha = 36.87^\circ$$

$$3\sin 2x + 4\cos 2x = 5 \cos(2x - 36.87^\circ)$$



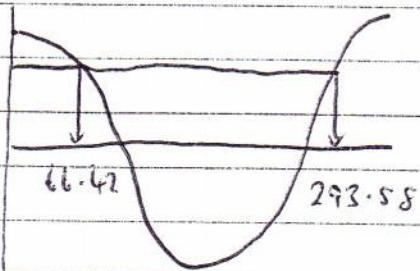
Question 6 continued

$$\text{d) } 4\cos 2x + 3\sin 2x = 2$$

$$5 \cos(2x - 36.87) = 2$$

$$\cos(2x - 36.87) = \frac{2}{5}$$

$$2x - 36.87 = 66.42^\circ$$



$$\text{When } 2x - 36.87 = 66.42$$

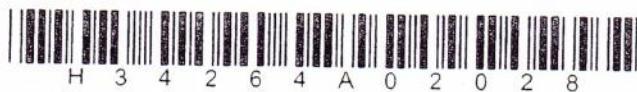
$$x = 51.65^\circ$$

$$\underline{x = 51.7^\circ}$$

and

$$2x - 36.87 = 293.58^\circ$$

$$\underline{x = 165.2^\circ}$$



8. (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8 \cos x = 0$$

giving your answers to 2 decimal places.

(5)

a) $\sin 2x = 2 \sin x \cos x$

(b) $\operatorname{cosec} x - 8 \cos x = 0$

$$\frac{1}{\sin x} - 8 \cos x = 0 \quad \text{MULTIPLY BY } \sin x$$

$$1 - 8 \sin x \cos x = 0$$

$$1 - 8 \sin x \cos x = 1$$

$$4(2 \sin x \cos x) = 1$$

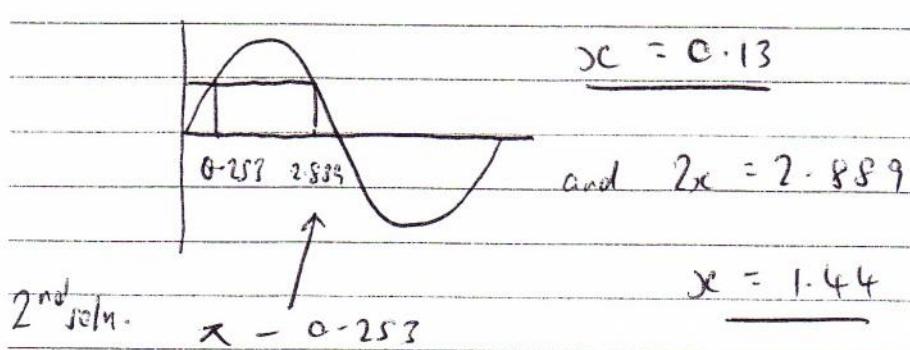
$$4(\sin 2x) = 1$$

$$\sin 2x = \frac{1}{4}$$

Radians

$$2x = 0.253$$

$$x = 0.13$$



1. (a) Show that

$$\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta. \quad (2)$$

(b) Hence find, for $-180^\circ \leq \theta < 180^\circ$, all the solutions of

$$\frac{2 \sin 2\theta}{1 + \cos 2\theta} = 1.$$

Give your answers to 1 decimal place.

(3)

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\text{so } \frac{2 \sin \theta \cos \theta}{1 + 2 \cos^2 \theta - 1}$$

$$= \frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta}$$

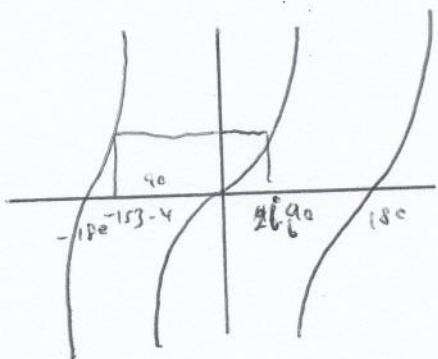
$$= \frac{\sin \theta}{\cos \theta} = \tan \theta.$$

b) $2 \tan \theta = 1$

~~$\theta = 45^\circ$~~ $\tan \theta = \frac{1}{2}$

$$\theta = 26.6^\circ, -153.4^\circ$$

$$(26.6^\circ - 180^\circ)$$



7. (a) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 4 decimal places.

(3)

- (b) (i) Find the maximum value of $2 \sin \theta - 1.5 \cos \theta$.

- (ii) Find the value of θ , for $0 \leq \theta < \pi$, at which this maximum occurs.

(3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2 \sin\left(\frac{4\pi t}{25}\right) - 1.5 \cos\left(\frac{4\pi t}{25}\right), \quad 0 \leq t < 12,$$

where t hours is the number of hours after midday.

- (c) Calculate the maximum value of H predicted by this model and the value of t , to 2 decimal places, when this maximum occurs.

(3)

- (d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres.

(6)

$$(7) \quad R^2 = 2^2 + 1.5^2 \quad R \sin(\theta - \alpha) = R \sin \theta \cos \alpha - R \cos \theta \sin \alpha$$

$$R = 2.5 \quad R \sin \theta \cos \alpha = 2 \sin \theta$$

$$\text{and } R \cos \theta \sin \alpha = 1.5 \cos \theta$$

$$R \cos \alpha = 2 \quad R \sin \alpha = 1.5$$

$$\frac{R \sin \alpha}{R \cos \alpha} = \frac{1.5}{2}$$

$$\tan \alpha = \frac{3}{4}$$

$$\alpha = 0.6435$$

$$\Rightarrow 2.5 \sin(\theta - 0.6435)$$

(b) (i)

$$2\sin\theta - 1.5\cos\theta = 2.5 \sin(\theta - 0.6435)$$

$$\text{MAX. value} = 2.5$$

$$\text{when } \sin(\theta - 0.6435) = 1$$

$$\theta - 0.6435 = \frac{\pi}{2}$$

$$\theta = 2.21$$

(c) $H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right)$

$$= 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right)$$

$$\begin{aligned}\text{MAX } H &= 6 + 2.5 \\ &= 8.5\text{m}\end{aligned}$$

$$\text{when } \sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1$$

$$\frac{4\pi t}{25} - 0.6435 = \frac{\pi}{2}$$

$$\frac{4\pi t}{25} = \frac{\pi}{2} + 0.6435$$

$$t = \frac{25}{4\pi} \left(\frac{\pi}{2} + 0.6435 \right)$$

$$\underline{t = 4.41}$$

(d)

$$6 + 2 \cdot 5 \sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7$$

$$2 \cdot 5 \sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1$$

$$\sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5}$$

$$\frac{4\pi t}{25} - 0.6435 = \sin^{-1}\left(\frac{1}{2.5}\right)$$

$$\frac{4\pi t}{25} - 0.6435 = 0.4115$$

$$\frac{4\pi t}{25} = 1.055$$

$$t = 2.1$$

Time 14.06

2 hrs

$$0.1 \times 60 \text{ minutes} \\ = 6 \text{ minutes}$$

2.

$$f(x) = 2 \sin(x^2) + x - 2, \quad 0 \leq x < 2\pi$$

- (a) Show that $f(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$

(2)

The equation $f(x) = 0$ can be written as $x = [\arcsin(1 - 0.5x)]^{\frac{1}{2}}$.

- (b) Use the iterative formula

$$x_{n+1} = [\arcsin(1 - 0.5x_n)]^{\frac{1}{2}}, \quad x_0 = 0.8$$

to find the values of x_1 , x_2 and x_3 , giving your answers to 5 decimal places.

(3)

- (c) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

(3)

a) $f(0.75) = -0.18339\dots$
 $f(0.85) = 0.1725$

as there is a change of sign

$$0.75 < \alpha < 0.85$$

b) REMEMBER \arcsin is \sin^{-1}

$$x_0 = 0.8$$

$$x_1 = [\sin^{-1}(1 - 0.5 \times 0.8)]^{\frac{1}{2}} = 0.80219 \text{ (5dp)}$$

$$x_2 = 0.80133 \text{ (5dp)}$$

$$x_3 = 0.80167 \text{ (5dp)}$$

c) $f(0.801565) = 2 \sin(0.801565)^2 + x - 2$
 $= -2.704 \times 10^{-5} < 0$

$$f(0.801575) = 8.6205 \times 10^{-6} > 0$$

Change of sign

so α between

0.801565 and 0.801575

$$\therefore \alpha = 0.80157 \text{ (5dp)}$$



6. (a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan \theta, \quad \theta \neq 90n^\circ, n \in \mathbb{Z} \quad (4)$$

(b) Hence, or otherwise,

(i) show that $\tan 15^\circ = 2 - \sqrt{3}$, (3)

(ii) solve, for $0 < x < 360^\circ$,

$$\operatorname{cosec} 4x - \cot 4x = 1$$

(5)

$$a) \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \frac{1 - \cos 2\theta}{\sin 2\theta}$$

$$= \frac{1 - (1 - 2\sin^2\theta)}{2 \sin \theta \cos \theta}$$

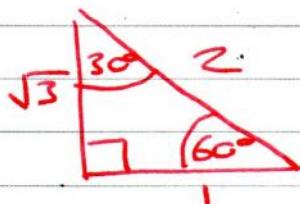
$$= \frac{2\sin^2\theta}{2 \sin \theta \cos \theta} = \tan \theta \quad \text{as required}$$

$$b) (i) \text{ let } \theta = 15^\circ$$

$$\therefore \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \frac{1}{\sin 30^\circ} - \frac{1}{\tan 30^\circ}$$

$$= \frac{1}{\frac{1}{2}} -$$

$$\frac{1}{\frac{1}{\sqrt{3}}}$$



$$= 2 - \sqrt{3} \quad (\text{as required}) \quad \begin{aligned} \sin 30^\circ &= \frac{1}{2} \\ \tan 30^\circ &= \frac{1}{\sqrt{3}} \end{aligned}$$



Question 6 continued

$$(ii) \csc 4x - \cot 4x = 1$$

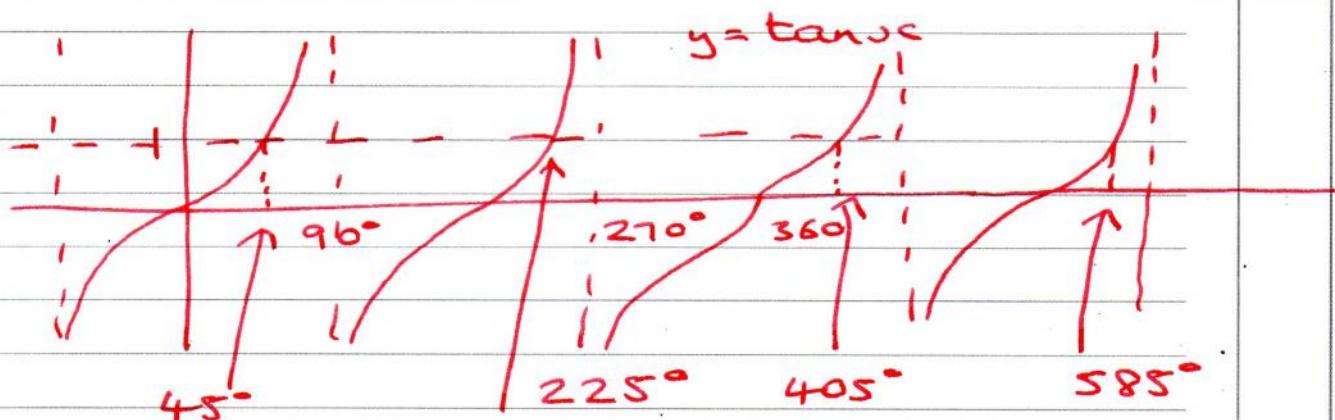
$$\frac{1}{\sin 4x} - \frac{\cos 4x}{\sin 4x} = 1$$

looking at original equation

$$\text{then } \theta = 2x$$

$$\therefore \tan 2x = 1$$

$$2x = \tan^{-1}(1)$$



$$\text{for } 0 < x < 360^\circ$$

$$0 < 2x < 720^\circ$$

$$\therefore 2x = 45^\circ, 225^\circ, 405^\circ, 585^\circ$$

$$\therefore x = 22.5^\circ, 112.5^\circ, 202.5^\circ, 292.5^\circ$$



8. (a) Express $2\cos 3x - 3\sin 3x$ in the form $R \cos(3x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures. (4)

$$f(x) = e^{2x} \cos 3x$$

- (b) Show that $f'(x)$ can be written in the form

$$f'(x) = R e^{2x} \cos(3x + \alpha)$$

where R and α are the constants found in part (a). (5)

- (c) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point. (3)

a) $R \cos(3x + \alpha) = R \cos 3x \cos \alpha - R \sin 3x \sin \alpha$

using double angles

$$\frac{R \cos 3x \cos \alpha}{2 \cos 3x} - \frac{R \sin 3x \sin \alpha}{3 \sin 3x}$$

using above $R \cos \alpha = 2$
 $R \sin \alpha = 3$

$$\therefore \frac{R \sin \alpha}{R \cos \alpha} = \tan \alpha = \frac{3}{2}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{3}{2}\right)$$

$$\alpha = 0.9827937$$

$$\alpha = 0.983^\circ \text{ (3 sf)}$$

$$R = \sqrt{3^2 + 2^2} = \sqrt{13} = 3.60555$$

$$R = 3.61 \text{ (3 sf)}$$



Question 8 continued

b) $f(x) = e^{2x} \cos 3x$

$$\frac{du}{dx} = 2e^{2x} \quad \frac{dv}{dx} = -3\sin 3x$$

$$\begin{aligned} f'(x) &= u \frac{dv}{dx} + v \frac{du}{dx} \\ f'(x) &= e^{2x} \cdot (-3\sin 3x) + 2e^{2x} \cos 3x \\ &= e^{2x} (-3\sin 3x + 2\cos 3x) \\ &= e^{2x} (\underbrace{2\cos 3x - 3\sin 3x}_P) \end{aligned}$$

in part a) we worked out

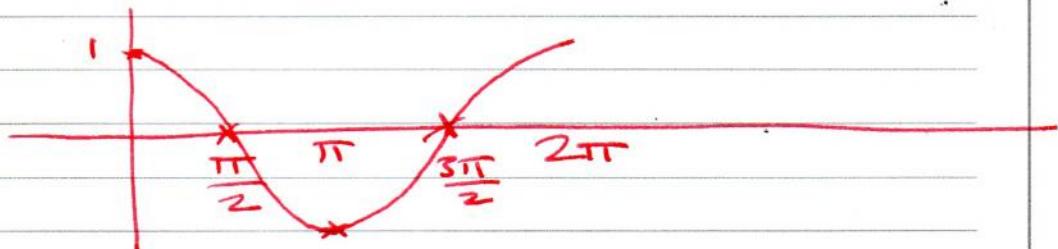
$$= e^{2x} (\sqrt{13} \cos(3x + 0.983))$$

c) We have turning point when
 $f'(x) = 0$

$$0 = e^{2x} \times \sqrt{13} \times \cos(3x + 0.983)$$

$\sqrt{13} e^{2x}$ cannot = 0

$$\therefore \cos(3x + 0.983) = 0$$



$$0 < x < \frac{\pi}{2}$$

$$0 < 3x + 0.983 < \frac{3\pi}{2} + 0.983$$

Solutions are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$



5. (a) Express $4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta$ in terms of $\sin \theta$ and $\cos \theta$.

(2)

- (b) Hence show that

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = \sec^2 \theta$$

(4)

- (c) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = 4$$

giving your answers in terms of π .

(3)

$$(a) \frac{4}{\sin^2 2\theta} - \frac{1}{\sin^2 \theta}$$

$$= \frac{4}{(2\sin \theta \cos \theta)^2} - \frac{1}{\sin^2 \theta}$$

$$= \frac{4}{4\sin^2 \theta \cos^2 \theta} - \frac{1}{\sin^2 \theta}$$

$$(b) = \frac{1 - \cos^2 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$= \frac{\sin^2 \theta}{\sin^2 \theta \cos^2 \theta}$$

$$= \sec^2 \theta$$



Question 5 continued

$$(c) \sec^2 \theta = 4$$

$$\frac{1}{\cos^2 \theta} = 4$$

$$\cos^2 \theta = \frac{1}{4}$$

$$\cos \theta = \frac{1}{2} \quad \text{and} \quad \cos \theta = -\frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

$$\theta = 2\frac{\pi}{3}$$



8.

$$f(x) = 7 \cos 2x - 24 \sin 2x$$

Given that $f(x) = R \cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$,

- (a) find the value of R and the value of α .

(3)

- (b) Hence solve the equation

$$7 \cos 2x - 24 \sin 2x = 12.5$$

for $0 \leq x < 180^\circ$, giving your answers to 1 decimal place.

(5)

- (c) Express $14 \cos^2 x - 48 \sin x \cos x$ in the form $a \cos 2x + b \sin 2x + c$, where a , b , and c are constants to be found.

(2)

- (d) Hence, using your answers to parts (a) and (c), deduce the maximum value of

$$14 \cos^2 x - 48 \sin x \cos x$$

(2)

8) $R \cos(2x + \alpha) = R \cos 2x \cos \alpha - R \sin 2x \sin \alpha$
 $f(x) = 7 \cos 2x - 24 \sin 2x$

$$\frac{R \sin \alpha}{R \cos \alpha} = \frac{-24}{7}$$

$$\tan \alpha = \frac{24}{7}$$

$$\alpha = 73.74^\circ$$

$$R = \sqrt{24^2 + 7^2}$$

$$R = 25$$

$$f(x) = 25 \cos(2x + 73.74^\circ)$$

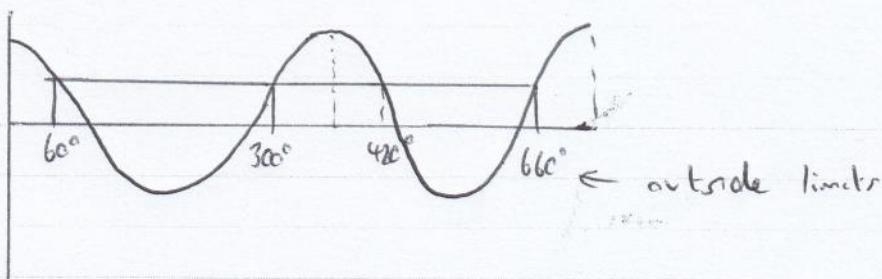


Question 8 continued

$$\text{b) } 25 \cos(2x + 73.74) = 12.5$$

$$\cos(2x + 73.74) = \frac{1}{2}$$

$$2x + 73.74 = 60^\circ$$



$$2x + 73.74 = 60^\circ, 300^\circ, 420^\circ$$

$$2x = -13.74, 226.26, 346.26^\circ$$

$$x = 173.1^\circ, 113.1^\circ$$

(c)

$$14 \cos^2 x - 48 \sin x \cos x$$

$$= 14 \left(\frac{\cos 2x + 1}{2} \right) - 24 \sin 2x$$

$$= 7(\cos 2x + 1) - 24 \sin 2x$$

$$= 7 \cos 2x + 7 - 24 \sin 2x$$



Question 8 continued

8 d)

$$\begin{aligned} & 14 \cos^2 x - 48 \sin x \cos x \\ &= 7 \cos 2x - 24 \sin 2x + 7 \\ &= 25 \cos(2x + 73.74) + 7 \\ \text{MAX.} &= 25 + 7 \\ &= \underline{\underline{32}}. \end{aligned}$$



4. (a) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(b) $p(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \quad 0 \leq \theta \leq 2\pi$

Calculate

(i) the maximum value of $p(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)

a) $6\cos\theta + 8\sin\theta = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

so $R\cos\alpha = 6$ and $R\sin\alpha = 8$

$$\tan\alpha = \frac{8}{6} \quad (\text{calculator in radians})$$

$$\alpha = \tan^{-1}\left(\frac{8}{6}\right) = 0.9272952^\circ$$

$$\alpha = 0.927^\circ \quad (3 \text{ dp})$$

b) $p(\theta) = \frac{4}{12 + (6\cos\theta + 8\sin\theta)}$

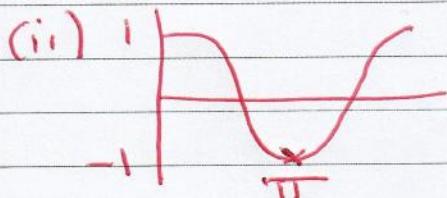
$$\begin{aligned} R^2 &= 8^2 + 6^2 \\ R^2 &= 100 \end{aligned} \quad \begin{aligned} 6\cos\theta + 8\sin\theta &= R\cos(\theta - \alpha) \\ &= 10\cos(\theta - 0.927) \end{aligned}$$

$$\begin{aligned} R &= 10 \\ \text{For max } p(\theta) &\text{ want } 10\cos(\theta - 0.927) \text{ as small as possible} \end{aligned}$$

min of cos is -1

$$\text{so min } 10\cos(\theta - 0.927) = -10$$

$$p(\theta) = \frac{4}{12 - 10} = \frac{4}{2} = \underline{\underline{2}}$$



max occurs when

$$\theta - 0.927 = \pi$$

$$\theta = \pi + 0.927$$

$$\theta = 4.0685927$$

$$\theta = 4.07 \quad (2 \text{ dp})$$



6. (i) Without using a calculator, find the exact value of

$$(\sin 22.5^\circ + \cos 22.5^\circ)^2$$

You must show each stage of your working.

(5)

- (ii) (a) Show that $\cos 2\theta + \sin \theta = 1$ may be written in the form

$$k \sin^2 \theta - \sin \theta = 0, \text{ stating the value of } k.$$

(2)

- (b) Hence solve, for $0 \leq \theta < 360^\circ$, the equation

$$\cos 2\theta + \sin \theta = 1$$

(i) $(\sin 22.5^\circ + \cos 22.5^\circ)^2$ (4)

$$= \sin^2 22.5^\circ + 2 \sin 22.5^\circ \cos 22.5^\circ$$

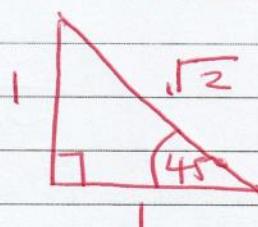
$$+ \cos^2 22.5^\circ$$

$$= (\sin^2 22.5^\circ + \cos^2 22.5^\circ) + 2 \sin 22.5^\circ \cos 22.5^\circ$$

$$\begin{array}{l} \text{sin}^2 \theta + \text{cos}^2 \theta = 1 \\ \hline \end{array}$$

$$= 1 + \sin 45^\circ$$

$$= 1 + \frac{\sqrt{2}}{2}$$



$$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

(iii) a) $\cos 2\theta + \sin \theta = 1$

$$1 - 2 \sin^2 \theta + \sin \theta = 1$$

$$0 = 2 \sin^2 \theta - \sin \theta$$

$$\text{where } k = 2$$

$$\left. \begin{array}{l} \cos 2\theta = 1 - 2 \sin^2 \theta \\ \hline \end{array} \right\}$$

b) $\sin(2 \sin \theta - 1) = 0$

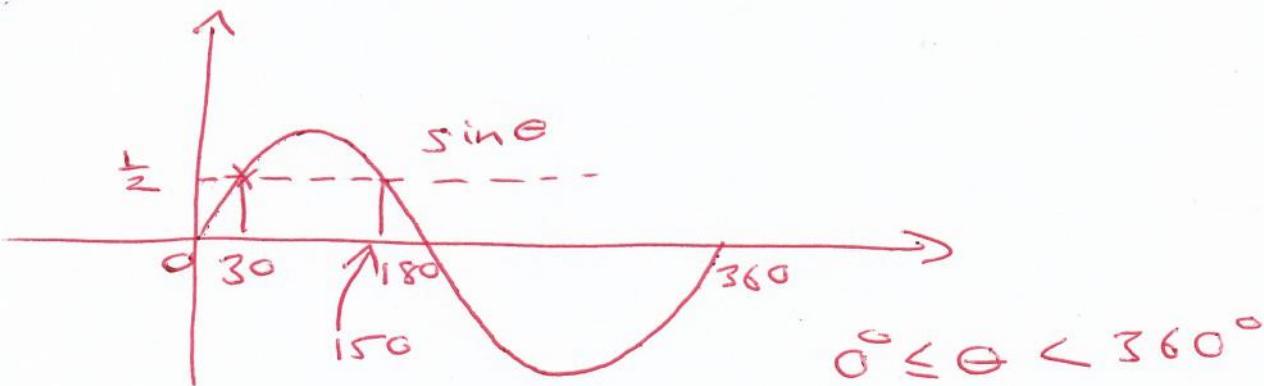
Either $\sin \theta = 0$ or $2 \sin \theta - 1 = 0$

$$\sin \theta = \frac{1}{2}$$



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6(ii) b)
continued



$$0^\circ \leq \theta < 360^\circ$$

$$\sin \theta = 0, \quad \theta = \underline{0^\circ, 180^\circ}$$

$$\sin \theta = \frac{1}{2}, \quad \theta = \underline{30^\circ, 150^\circ}$$

3. Given that

$$2\cos(x + 50)^\circ = \sin(x + 40)^\circ$$

(a) Show, without using a calculator, that

$$\tan x^\circ = \frac{1}{3} \tan 40^\circ \quad (4)$$

(b) Hence solve, for $0 \leq \theta < 360$,

$$2\cos(2\theta + 50)^\circ = \sin(2\theta + 40)^\circ$$

giving your answers to 1 decimal place.

(4)

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\therefore 2 \cos x \cos 50^\circ - 2 \sin x \sin 50^\circ \\ = \sin x \cos 40^\circ + \cos x \sin 40^\circ$$

÷ through by $\cos x$ to get $\tan x$

$$2 \frac{\cos x \cos 50^\circ}{\cos x} - 2 \frac{\sin x \sin 50^\circ}{\cos x} \\ = \frac{\sin x \cos 40^\circ}{\cos x} + \frac{\cos x \sin 40^\circ}{\cos x}$$

$$2 \cos 50^\circ - 2 \tan x \sin 50^\circ \\ = \tan x \cos 40^\circ + \sin 40^\circ$$

$$\text{but } \cos 50^\circ = \sin 40^\circ$$

$$\text{and } \cos 40^\circ = \sin 50^\circ$$

(change all 50° to 40°)

$$2 \sin 40^\circ - 2 \tan x \cos 40^\circ \\ = \tan x \cos 40^\circ + \sin 40^\circ$$

÷ through by $\cos 40^\circ$



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3a continued

$$2 \frac{\sin 40^\circ}{\cos 40^\circ} - 2 \tan x \frac{\cos 40^\circ}{\cos 40^\circ}$$

$$= \tan x \frac{\cos 40^\circ}{\cos 40^\circ} + \frac{\sin 40^\circ}{\cos 40^\circ}$$

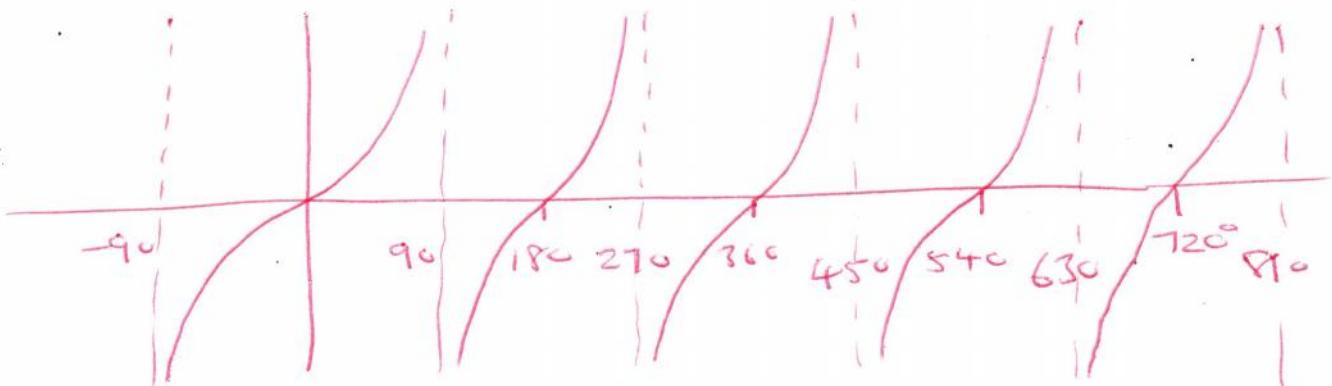
$$2 \tan 40^\circ - 2 \tan x = \tan x + \tan 40^\circ$$

$$2 \tan 40^\circ - \tan 40^\circ = \tan x + 2 \tan x$$

$$\tan 40^\circ = 3 \tan x$$

$$\frac{1}{3} \tan 40^\circ = \tan x$$

b) let $x = 2\theta$



$$2\theta = \tan^{-1} \frac{1}{3} \tan 40^\circ$$

$$2\theta = 15.6263^\circ$$

$$180 + 15.6263 = 195.6263^\circ$$

$$360 + 15.6263 = 375.6263^\circ$$

$$540 + 15.6263 = 555.6263^\circ$$

$$720 + 15.6263 = 735.6263^\circ$$

$$\theta = 7.81315^\circ, 97.81315^\circ, 187.81315^\circ,$$

$$277.81315^\circ, 367.81315^\circ$$

$$\theta = 7.8^\circ, 97.8^\circ, 187.8^\circ, 277.8^\circ$$

Out of range

8.

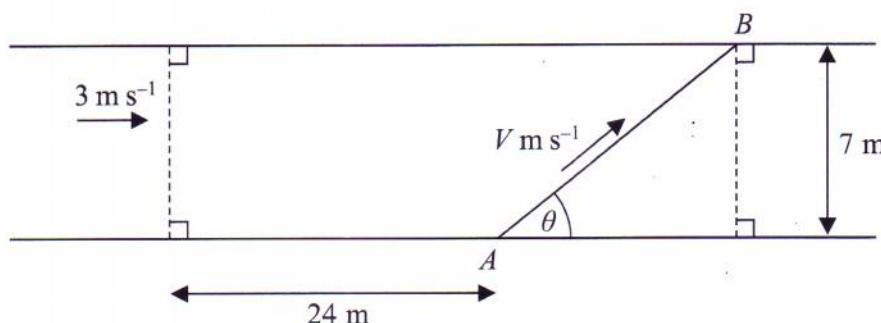


Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1} .

Kate is 24 m ahead of John when she starts to cross the road from the fixed point A .

John passes her as she reaches the other side of the road at a variable point B , as shown in Figure 2.

Kate's speed is $V \text{ m s}^{-1}$ and she moves in a straight line, which makes an angle θ , $0 < \theta < 150^\circ$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24 \sin \theta + 7 \cos \theta}, \quad 0 < \theta < 150^\circ$$

- (a) Express $24 \sin \theta + 7 \cos \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places.

(3)

Given that θ varies,

- (b) find the minimum value of V .

(2)

Given that Kate's speed has the value found in part (b),

- (c) find the distance AB .

(3)

Given instead that Kate's speed is 1.68 m s^{-1} ,

- (d) find the two possible values of the angle θ , given that $0 < \theta < 150^\circ$.

(6)

a) $R \cos(\theta - \alpha) = R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$
 $= 7 \cos \theta + 24 \sin \theta$

$R \cos \alpha = 7$, $R \sin \alpha = 24$

$\frac{R \sin \alpha}{R \cos \alpha} = \frac{24}{7}$, $\tan \alpha = \frac{24}{7}$ $\alpha = 73.7397^\circ$

$\alpha = 73.74^\circ \text{ (2dp)}$

$R = \sqrt{7^2 + 24^2} = 25$



$\therefore 24 \sin \theta + 7 \cos \theta = 25 \cos(\theta - 73.74)$

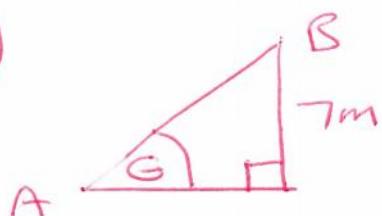
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8b) $V_{\min} = \frac{21}{\max(25 \cos(\theta - 73.74))}$

$\nwarrow \max \cos = 1$

$$V_{\min} = \frac{21}{\underline{\underline{25}}}$$

c)



from b) max when

$$\cos(\theta - 73.74) = 1$$

$$\text{but } \cos 0^\circ = 1$$

$$\therefore \theta = 73.74$$

$$\sin 73.74^\circ = \frac{7}{AB}$$

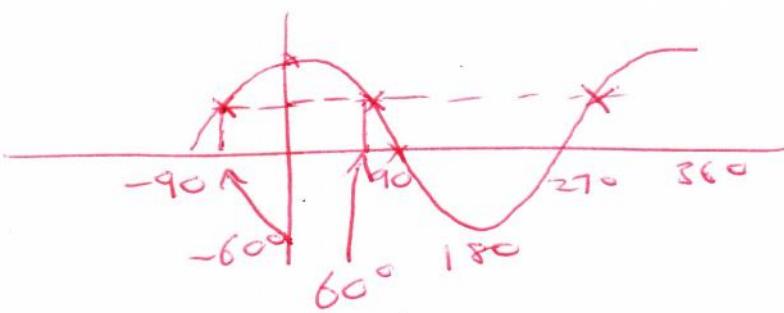
$$AB = \frac{7}{\sin 73.74^\circ} = \frac{7}{\underline{\underline{\sin 73.74^\circ}}} = 7.2916591 \\ = 7.29 \text{ m (3sf)}$$

d) $\frac{21}{25 \cos(\theta - 73.74)} = 1.68$

$$\cos(\theta - 73.74) = \frac{21}{25 \times 1.68} = 0.5$$

$$0 < \theta < 180^\circ \quad \theta = \cos^{-1} 0.5 = 60^\circ$$

$$-73.74 < \theta - 73.74 < 76.26$$



values in range

$$\theta - 73.74 = -60, 60$$

$$\theta = -60 + 73.74 = \underline{\underline{13.74^\circ}}$$

$$\theta = 60 + 73.74 = \underline{\underline{133.74^\circ}}$$