C3 Jan 2008

8. The functions f and g are defined by

$$\mathrm{f}: x \mapsto 1 - 2x^3, \; x \in \mathbb{R}$$

$$\mathrm{g}: x \mapsto \frac{3}{x} - 4, \; x > 0, \; x \in \mathbb{R}$$

(a) Find the inverse function f^{-1}.

(2)

(b) Show that the composite function gf is

$$\mathrm{gf}: x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(c) Solve $\mathrm{gf}(x) = 0$.

(2)

(d) Use calculus to find the coordinates of the stationary point on the graph of $y = \mathrm{gf}(x)$.

(5)

C3 Jan 2009

Leave blank

5. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R}$$
$$\mathrm{g} : x \mapsto \mathrm{e}^{x^2}, \quad x \in \mathbb{R}$$

(a) Write down the range of g.

(1)

(b) Show that the composite function fg is defined by

$$\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}, \quad x \in \mathbb{R}.$$

(2)

(c) Write down the range of fg.

(1)

(d) Solve the equation $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{fg}(x)\right] = x(x\mathrm{e}^{x^2} + 2)$.

(6)

C3 Jan 2011

Leave blank

6. The function f is defined by

$$\text{f}: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R},\ x \neq 5$$

(a) Find $\text{f}^{-1}(x)$.

(3)

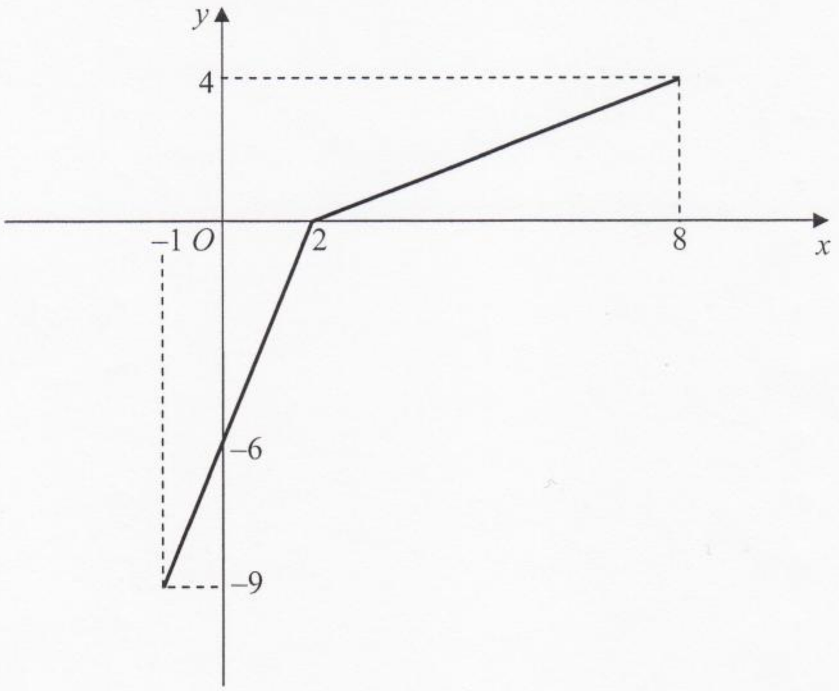

Figure 2

The function g has domain $-1 \leqslant x \leqslant 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = \text{g}(x)$.

(b) Write down the range of g.

(1)

(c) Find gg(2).

(2)

(d) Find fg(8).

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |\text{g}(x)|$,

(ii) $y = \text{g}^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1}.

(1)

C3 Jan 2012

Leave blank

7. The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}, \qquad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $f(x) = \dfrac{1}{2x-1}$

(4)

(b) Find $f^{-1}(x)$

(3)

(c) Find the domain of f^{-1}

(1)

$$g(x) = \ln(x+1)$$

(d) Find the solution of $fg(x) = \dfrac{1}{7}$, giving your answer in terms of e.

(4)

C3 June 2007

Leave blank

5. The functions f and g are defined by

$$f : x \mapsto \ln(2x-1), \qquad x \in \mathbb{R},\ x > \frac{1}{2},$$

$$g : x \mapsto \frac{2}{x-3}, \qquad x \in \mathbb{R},\ x \neq 3.$$

(a) Find the exact value of fg(4).
(2)

(b) Find the inverse function $f^{-1}(x)$, stating its domain.
(4)

(c) Sketch the graph of $y = |g(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the y-axis.
(3)

(d) Find the exact values of x for which $\left|\frac{2}{x-3}\right| = 3$.
(3)

C3 June 2008

Leave blank

4. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $\mathrm{f}(x) = \dfrac{1}{x+1}$, $\quad x > 3$.
(4)

(b) Find the range of f.
(2)

(c) Find $\mathrm{f}^{-1}(x)$. State the domain of this inverse function.
(3)

The function g is defined by

$$\mathrm{g}: x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $\mathrm{fg}(x) = \dfrac{1}{8}$.
(3)

C3 June 2009

Leave blank

7. The function f is defined by

$$\mathrm{f}(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2$$

(a) Show that $\mathrm{f}(x) = \dfrac{x-3}{x-2}$ **(5)**

The function g is defined by

$$\mathrm{g}(x) = \frac{\mathrm{e}^x - 3}{\mathrm{e}^x - 2}, \quad x \in \mathbb{R},\ x \neq \ln 2$$

(b) Differentiate $\mathrm{g}(x)$ to show that $\mathrm{g}'(x) = \dfrac{\mathrm{e}^x}{(\mathrm{e}^x - 2)^2}$ **(3)**

(c) Find the exact values of x for which $\mathrm{g}'(x) = 1$ **(4)**

C3 June 2010

Leave blank

4. The function f is defined by

$$\mathrm{f}: x \mapsto |2x-5|, \quad x \in \mathbb{R}$$

(a) Sketch the graph with equation $y = \mathrm{f}(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(b) Solve $\mathrm{f}(x) = 15 + x$.

(3)

The function g is defined by

$$\mathrm{g}: x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leqslant x \leqslant 5$$

(c) Find $\mathrm{fg}(2)$.

(2)

(d) Find the range of g.

(3)

C3 JUNE 2011

Leave blank

4. The function f is defined by

$$f : x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R},\ x \geqslant -1$$

(a) Find $f^{-1}(x)$.

(3)

(b) Find the domain of f^{-1}.

(1)

The function g is defined by

$$g : x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $fg(x)$, giving your answer in its simplest form.

(3)

(d) Find the range of fg.

(1)

C3 June 2012

Leave blank

6. The functions f and g are defined by

$$f : x \mapsto e^x + 2, \qquad x \in \mathbb{R}$$

$$g : x \mapsto \ln x, \qquad x > 0$$

(a) State the range of f.

(1)

(b) Find $fg(x)$, giving your answer in its simplest form.

(2)

(c) Find the exact value of x for which $f(2x+3) = 6$

(4)

(d) Find f^{-1}, the inverse function of f, stating its domain.

(3)

(e) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.

(4)

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7.
$$\mathrm{h}(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $\mathrm{h}(x) = \dfrac{2x}{x^2+5}$ **(4)**

(b) Hence, or otherwise, find $\mathrm{h}'(x)$ in its simplest form. **(3)**

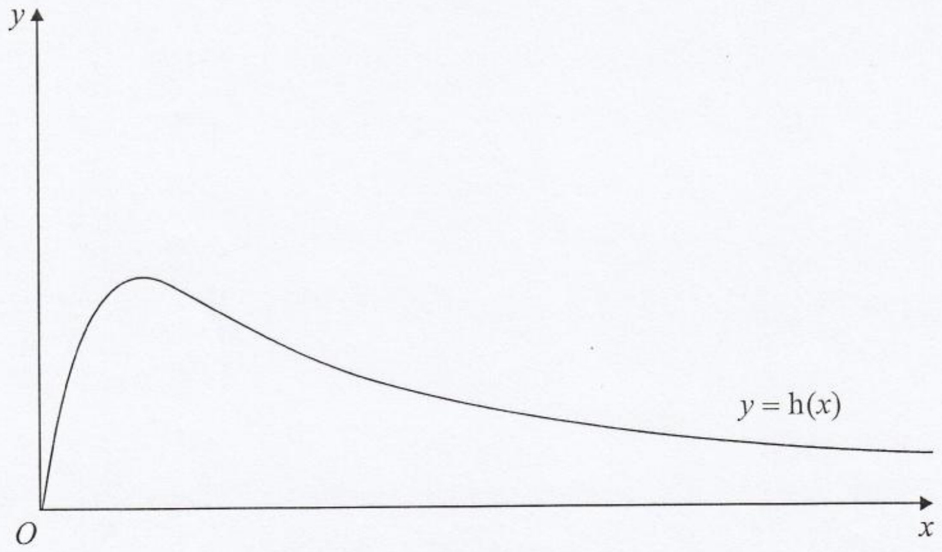

Figure 2

Figure 2 shows a graph of the curve with equation $y = \mathrm{h}(x)$.

(c) Calculate the range of $\mathrm{h}(x)$. **(5)**

C3 June 2013

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7. The function f has domain $-2 \leqslant x \leqslant 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = \mathrm{f}(x)$ is shown in Figure 1.

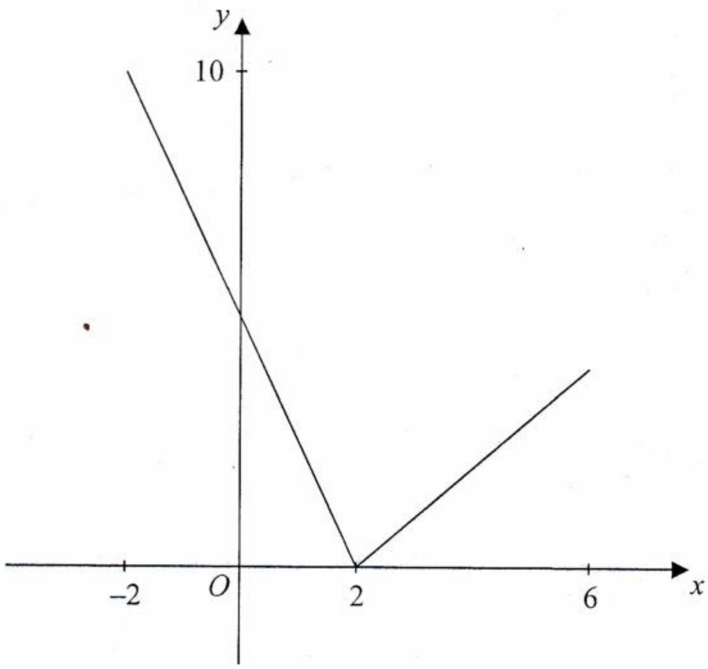

Figure 1

(a) Write down the range of f.

(1)

(b) Find ff(0).

(2)

The function g is defined by

$$\mathrm{g} : x \rightarrow \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $\mathrm{g}^{-1}(x)$

(3)

(d) Solve the equation $\mathrm{gf}(x) = 16$

(5)

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