C3 Jan 2008

1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

Leave blank

2.

$$f(x) = \frac{2x+2}{x^2-2x-3} - \frac{x+1}{x-3}$$

(a) Express $f(x)$ as a single fraction in its simplest form.

(4)

(b) Hence show that $f'(x) = \dfrac{2}{(x-3)^2}$

(3)

C3 Jan 2010

Leave blank

1. Express

$$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$$

as a single fraction in its simplest form.

(4)

C3 Jan 2011

Leave blank

2. (a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \qquad x > 1,$$

(b) show that

$$f(x) = \frac{3}{2x-1}$$

(2)

(c) Hence differentiate $f(x)$ and find $f'(2)$.

(3)

C3 June 2007

Leave blank

2. $$f(x) = \frac{2x+3}{x+2} - \frac{9+2x}{2x^2+3x-2}, \qquad x > \frac{1}{2}.$$

(a) Show that $f(x) = \dfrac{4x-6}{2x-1}$.

(7)

(b) Hence, or otherwise, find $f'(x)$ in its simplest form.

(3)

C3 June 2010

Leave blank

8. (a) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$$

(3)

Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15), \quad x \neq -5,$$

(b) find x in terms of e.

(4)

Leave blank

7. $$\mathrm{f}(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3,\ x \neq -\frac{1}{2}$$

(a) Show that

$$\mathrm{f}(x) = \frac{5}{(2x+1)(x+3)}$$

(5)

The curve C has equation $y = \mathrm{f}(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(b) Find an equation of the normal to C at P.

(8)

C3 June 2012

Leave blank

1. Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

Leave blank

7. $$h(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $h(x) = \dfrac{2x}{x^2+5}$ **(4)**

(b) Hence, or otherwise, find $h'(x)$ in its simplest form. **(3)**

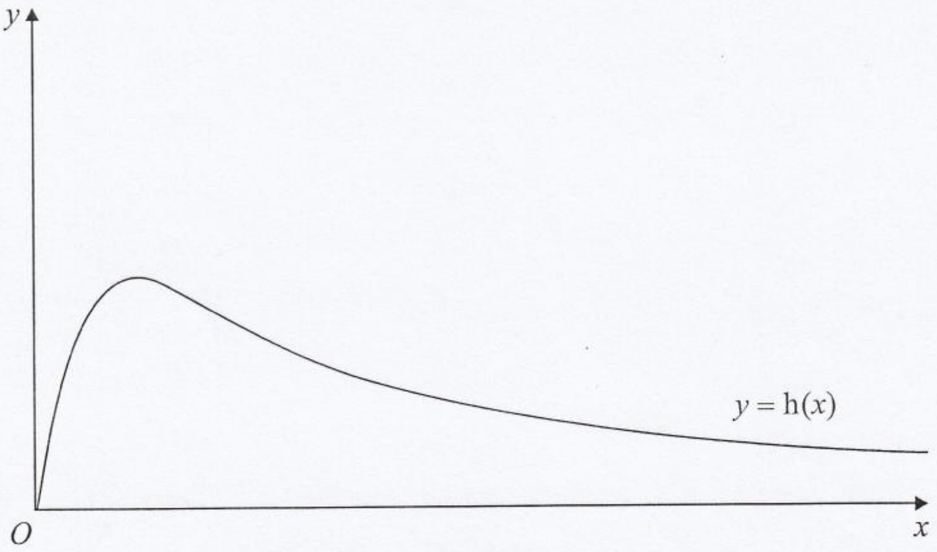

Figure 2

Figure 2 shows a graph of the curve with equation $y = h(x)$.

(c) Calculate the range of $h(x)$. **(5)**

C3 June 2013

Leave blank

1. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants a, b, c, d and e.

(4)