

1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e .

(4)

$$\begin{array}{r} 2x^2 - 1 \\ x^2 - 1 \overline{)2x^4 - 3x^2 + x + 1} \\ - 2x^4 - 2x^2 \\ \hline -x^2 + x + 1 \\ -x^2 \quad \quad \quad + 1 \\ \hline x \end{array}$$

$$A = 2$$

$$\equiv 2x^2 - 1 + \frac{x}{x^2 - 1} \quad B = 0$$

$$C = -1$$

$$D = 1$$

$$E = 0$$

~~Key~~



2.

$$f(x) = \frac{2x+2}{x^2 - 2x - 3} - \frac{x+1}{x-3}$$

(a) Express $f(x)$ as a single fraction in its simplest form.

(4)

(b) Hence show that $f'(x) = \frac{2}{(x-3)^2}$

(3)

$$f(x) = \frac{2x+2}{(x-3)(x+1)} - \frac{x+1}{x-3}$$

$$= \frac{(2x+2) - (x+1)(x+1)}{(x-3)(x+1)}$$

$$= \frac{2(x+1) - (x+1)(x+1)}{(x-3)(x+1)}$$

$$= \frac{(x+1)(2-x-1)}{(x-3)(x+1)} \quad \leftarrow (\text{Factorise taking out } (x+1))$$

$$= \frac{(x+1)(1-x)}{(x-3)(x+1)}$$

$$= \frac{1-x}{x-3}$$

b) Quotient Rule

$$u = 1-x$$

$$v = x-3$$

$$\frac{du}{dx} = -1$$

$$\frac{dv}{dx} = 1$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \frac{(x-3)(-1) - (1-x)(1)}{(x-3)^2}$$

$$= \frac{-x+3-1+x}{(x-3)^2} = \frac{2}{(x-3)^2}$$



1. Express

$$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$$

as a single fraction in its simplest form.

Factorise denominator

(4)

$$\frac{x+1}{3(x^2-1)} - \frac{1}{3x+1}$$

(difference of squares)

$$= \frac{x+1}{3(x+1)(x-1)} - \frac{1}{3x+1}$$

$$= \frac{1}{3(x-1)} - \frac{1}{3x+1}$$

Common

denominator $= \frac{3x+1 - 3(x-1)}{3(x-1)(3x+1)}$

$$= \frac{4}{3(x-1)(3x+1)}$$



2. (a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \quad x > 1,$$

- (b) show that

$$f(x) = \frac{3}{2x-1}.$$

(2)

- (c) Hence differentiate $f(x)$ and find $f'(2)$.

(3)

$$a) \quad \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} = \frac{(4x-1)(2x-1) - 3}{2(x-1)(2x-1)}$$

$$= \frac{8x^2 - 6x + 1 - 3}{2(x-1)(2x-1)}$$

$$= \frac{8x^2 - 6x - 2}{2(x-1)(2x-1)}$$

$$= \frac{2(4x^2 - 3x - 1)}{2(x-1)(2x-1)}$$

$$= \frac{x(x-1)(4x+1)}{2(x-1)(2x-1)}$$

$$= \frac{4x+1}{2x-1}$$

$$(b) f(x) = \frac{4x+1}{2x-1} - 2$$

$$= \frac{(4x+1) - 2(2x-1)}{2x-1}$$

$$= \frac{4x+1 - 4x+2}{2x-1}$$

$$= \frac{3}{2x-1}$$

$$(c) f(x) = 3(2x-1)^{-1}$$

$$\begin{aligned}f'(x) &= -3(2x-1)^{-2} \cdot 2 \\&= -6(2x-1)^{-2}\end{aligned}$$

$$\begin{aligned}f'(2) &= -6(2 \times 2 - 1)^{-2} \\&= -\frac{2}{3}\end{aligned}$$

2.

$$f(x) = \frac{2x+3}{x+2} - \frac{9+2x}{2x^2+3x-2}, \quad x > \frac{1}{2}.$$

(a) Show that $f(x) = \frac{4x-6}{2x-1}$. (7)

(b) Hence, or otherwise, find $f'(x)$ in its simplest form. (3)

$$f(x) = \frac{2x+3}{(x+2)} - \frac{9+2x}{(2x-1)(x+2)}$$

$$= \frac{(2x+3)(2x-1) - 9+2x}{(2x-1)(x+2)}$$

$$= \frac{4x^2 + 4x - 3 - 9 + 2x}{(2x-1)(x+2)}$$

$$= \frac{4x^2 + 6x - 12}{(2x-1)(x+2)}$$

$$= \frac{\cancel{(2x+1)}(4x-6)}{(2x-1)\cancel{(x+2)}}$$

$$= \frac{4x-6}{2x-1}$$

b) Quotient rule : $(u = 4x-6) \quad (v = 2x-1)$

$$\frac{du}{dx} = 4 \quad \frac{dv}{dx} = 2$$

$$f'(x) =$$

$$\frac{4(2x-1) - 2(4x-6)}{(2x-1)^2}$$

$$= \frac{8}{(2x-1)^2}$$



8. (a) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}.$$

(3)

Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15), \quad x \neq -5,$$

(b) find x in terms of e.

(4)

TOTAL FOR PAPER: 75 MARKS

END

$$(a) \quad \frac{(2x-1)(x+5)}{(x-3)(x+5)}$$
$$= \frac{2x-1}{x-3}$$

$$(b) \quad \ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15)$$

$$\ln(2x^2 + 9x - 5) - \ln(x^2 + 2x - 15) = 1$$

$$\ln\left(\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$$

$$\ln\left(\frac{2x-1}{x-3}\right) = 1$$

$$\frac{2x-1}{x-3} = e$$

$$2x-1 = e(x-3)$$

$$2x - 1 = ex - 3e$$

$$2x - ex = 1 - 3e$$

$$x(2 - e) = 1 - 3e$$

$$x = \frac{1 - 3e}{2 - e}$$

7. $f(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \quad x \neq \pm 3, x \neq -\frac{1}{2}$

(a) Show that

$$f(x) = \frac{5}{(2x+1)(x+3)} \quad (5)$$

The curve C has equation $y=f(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C .

(b) Find an equation of the normal to C at P . (8)

$$\begin{aligned} a) \quad & \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9} \\ &= \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{(x+3)(x-3)} \\ &= \frac{(4x-5)(x+3) - 2x(2x+1)}{(2x+1)(x-3)(x+3)} \\ &= \frac{4x^2 + 12x - 5x - 15 - 4x^2 - 2x}{(2x+1)(x-3)(x+3)} \\ &= \frac{5x - 15}{(2x+1)(x-3)(x+3)} \\ &= \frac{5(x-3)}{(2x+1)(x-3)(x+3)} \\ &= \frac{5}{(2x+1)(x+3)} \quad \text{as required} \end{aligned}$$



Question 7 continued

b) First find gradient of $f(x)$

$$\begin{aligned}y &= \frac{5}{(2x+1)(x+3)} = \frac{5}{2x^2+6x+x+3} \\&= \frac{5}{2x^2+7x+3} = 5(2x^2+7x+3)^{-1}\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= -5(2x^2+7x+3)^{-2} \times (4x+7) \\&= \frac{-5(4x+7)}{(2x^2+7x+3)^2}\end{aligned}$$

$$P(-1, -\frac{5}{2})$$

gradient at P, $x=-1$

$$\frac{dy}{dx} = \frac{-5(-4+7)}{(2(-1)^2-7+3)^2} = \frac{-15}{4}$$

\therefore gradient of normal = $\frac{4}{15}$

$$y - y_1 = m(x - x_1)$$

$$y - (-\frac{5}{2}) = \frac{4}{15}(x - -1)$$

$$y + \frac{5}{2} = \frac{4}{15}x + \frac{4}{15}$$

$$y + \frac{5}{2} = \frac{4}{15}(x+1) \text{ is the}$$

equation of normal
to C at P



1. Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

$$\frac{2(3x+2)}{(3x+2)(3x-2)} - \frac{2}{3x+1}$$

$$= \frac{2(3x+1) - 2(3x-2)}{(3x-2)(3x+1)}$$

$$= \frac{6x+2 - 6x+4}{(3x-2)(3x+1)}$$

$$= \frac{6}{(3x-2)(3x+1)}$$



7.

$$h(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \quad x \geq 0$$

(a) Show that $h(x) = \frac{2x}{x^2+5}$ (4)

(b) Hence, or otherwise, find $h'(x)$ in its simplest form. (3)



Figure 2

Figure 2 shows a graph of the curve with equation $y = h(x)$.

(c) Calculate the range of $h(x)$. (5)

$$\begin{aligned}
 a) \quad & h(x) = \frac{2(x^2+5) + 4(x+2) - 18}{(x^2+5)(x+2)} \\
 & = \frac{2x^2 + 10 + 4x + 8 - 18}{(x^2+5)(x+2)} \\
 & = \frac{2x^2 + 4x}{(x^2+5)(x+2)} = \frac{2x(x+2)}{(x^2+5)(x+2)} \\
 & = \frac{2x}{\underline{x^2+5}} \quad \text{as required}
 \end{aligned}$$



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7b) $h(x) = \frac{2x}{x^2 + 5}$

$$u = 2x \quad v = x^2 + 5$$

$$\frac{du}{dx} = 2 \quad \frac{dv}{dx} = 2x$$

$$h'(x) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \frac{2(x^2 + 5) - 2x \cdot 2x}{(x^2 + 5)^2}$$

$$h'(x) = \frac{2x^2 + 10 - 4x^2}{(x^2 + 5)^2} = \underline{\underline{-\frac{2x^2 + 10}{(x^2 + 5)^2}}}$$

c) Max point of curve when

$$h'(x) = 0$$

$$0 = \frac{-2x^2 + 10}{(x^2 + 5)^2}$$

$$-2x^2 + 10 = 0$$

$$2x^2 = 10$$

$$x^2 = 5$$

$$x = \sqrt{5} \quad \text{as } x \geq 0$$

don't need $-\sqrt{5}$

Minimum when $x = 0$, $h(x) = 0$

$$\text{Max when } x = \sqrt{5}, h(x) = \frac{2\sqrt{5}}{5+5}$$

$$= \frac{2\sqrt{5}}{10} = \frac{\sqrt{5}}{5}$$

$$\text{Range of } h(x) \quad 0 \leq y \leq \frac{\sqrt{5}}{5}$$

1. Given that

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} \equiv ax^2 + bx + c + \frac{dx + e}{x^2 - 4}, \quad x \neq \pm 2$$

find the values of the constants a, b, c, d and e .

(4)

$$\begin{array}{r}
 \overline{3x^2 - 2x + 7} \\
 x^2 - 4 \quad | \overline{3x^4 - 2x^3 - 5x^2 + 0x - 4} \\
 - \quad \overline{3x^4 + 0x^3 - 12x^2} \\
 \quad \quad \quad \downarrow \\
 \quad \quad \quad - 2x^3 + 7x^2 + 0x \\
 - \quad \overline{- 2x^3 + 8x} \\
 \quad \quad \quad \downarrow \\
 \quad \quad \quad 7x^2 - 8x - 4 \\
 - \quad \overline{7x^2} \\
 \quad \quad \quad - 8x - 24 \\
 - \quad \overline{- 8x + 24}
 \end{array}$$

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} = 3x^2 - 2x + 7 - \frac{8x + 24}{x^2 - 4}$$

$$\begin{aligned}
 \text{where } a &= 3 \\
 b &= -2 \\
 c &= 7 \\
 d &= -8 \\
 e &= 24
 \end{aligned}$$

